Competitive Trade: Ricardian Theory

By Kiminori Matsuyama

(Permanently) Work in Progress

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Overview

- Introduction
- Basic Ricardian Model (Two Countries, a Finite Number of Goods, Zero Trade Cost, Homothetic Preferences, Exogenous Technologies)
- A Ricardian Model with a Continuum of Goods
- Nontradeables, Trade Costs, and Globalization
- Non-Homothetic Preferences: Structural Change and North-South Trade
- Multi-Country Extensions and Bilateral Trade
- Bibliography

An extension to endogenize technological differences will be discussed in Part IV and Part V.

Introduction

General Theory of Competitive Trade (Part 1) highlighted the role of differences across countries.

- Countries trade and gain from it only when they differ in autarky prices.
- Law of Comparative Advantage is stated in terms of autarky price differences.

Hence, we may want to classify different models according to the differences assumed.

- ❖ Differences in Taste:
  - *Different cultures and preferences*: Japan exports chicken feet to China, because the Chinese love eating them, but the Japanese don't.
  - *Income differences with non-homothetic preferences*: US, EU, and Japan are the three biggest markets for SUV; China, India, and Indonesia are the three biggest markets for motorbikes.
- ❖ Differences in Technology:
  - *Climate and Geography*
  - *Technical Expertise*
- ❖ Differences in Factor Endowments
  - *Natural Resources*: Japan imports oil from Saudi Arabia

  - *Labor/Land Ratio:*
  - *Labor Force Compositions: Skilled/Unskilled*
- Differences in Policy and Institutions: Some countries may have tougher standards against pollution, child labor, etc., than others. Countries may differ in labor market flexibility, etc.

Empirically, these classifications are not always clear-cut. For example,

- Rich countries tend to have tougher environmental standards. Should we treat such policy differences as given? Or should we attribute them to income differences?
- Some countries have more educated labor forces than others. Should we treat them as factor endowment differences, or attribute them to the differences in cultures, educational systems, or some other related factors?
- Should we treat Japan's expertise in shipbuilding as given, or attribute it to its geography?
- Some anthropologists may want to attribute any cultural differences to its natural environments
- There may be some two-way causality; some differences across countries cause countries to trade, which in turn may amplify the differences across countries.

etc. etc.

For our purpose, we should treat these classifications merely as a way of identifying which difference the modeler has chosen to treat as exogenous.

Two Major Strands of Literature:

- **Ricardian Trade Theory (Part 2):**

  Technological Differences as the Basis of Trade

- **Factor Proportions Theory (Part 3):**

  Differences in Factor Endowment Compositions as the Basis of Trade

Ricardian Trade Theory:

- It treats the cross-country, cross-industry, technology differences as the basis of trade

- It abstracts from the roles of the cross-country differences in the factor endowments (proportions) and the cross-industry differences in the factor intensities

- It allows for simple characterization of the patterns of trade.

- It is well-suited to examine the effects of country (population) sizes, technology changes and transfers (because it abstracts from the roles of the factor endowments and factor intensities).

- It is relatively easy to allow for many goods (e.g., Dornbusch-Fischer-Samuelson, Itoh-Kiyono), many countries (e.g., Eaton-Kortum), many (i.e., non-representative) households (e.g., Flam-Helpman, Matsuyama), etc.

- It provides an important example where the Revenue function, R(p, V), is non-differentiable in p, so that x(p,V) jumps discretely.

Basic Ricardian Model (2 Countries, A Finite Number of Goods, Zero Trade Cost, Homothetic Preferences, Exogenous Technologies)

M = 1 (Nontradeable) Factor of Production (called Labor):

Endowment (L); Wage (w)

N (Tradeable) Commodities Produced

Outputs: $x = (x_1, x_2, ..., x_N)^T$
Output Prices: $p = (p_1, p_2, ..., p_N)$
GDP: $Y = px = wL = E$

Technology: Constant Returns to Scale & *No Joint Production*

$0 \le x_j \le L_j / a_j$ for j = 1, 2, …, N,

where L_j is the labor input in sector-j, and $a = (a_1, a_2, ..., a_N)$ is a N-dimensional row vector of the *unit labor requirement,* i.e., the inverse of the labor productivity.

Output and Revenue (GDP) Functions:

$x(p, L) \equiv \text{Argmax}_x \{px \mid ax \leq L;\ x \in R_+^N\}$;
$R(p, L) \equiv px(p,L) = \text{Max}_x \{px \mid ax \leq L;\ x \in R_+^N\}$.

Exercise: Derive $w = R_L(p,L)$ explicitly.

Exercise: Show that $R(p,L) = \text{Min}_w \{wL \mid a_j w \geq p_j \text{ for } j = 1, 2, \ldots, N\}$.
Hint: Consider the problem, $\mathcal{L}(w, x; L, p, a) = wL + \{p - wa\}x$, where $x = (x_1, x_2, \ldots, x_N)^T$ and x_j is the Lagrangian multiplier associated with the constraint, $a_j w \geq p_j$.
Intuitively, $x_j > 0$ requires that p_j/a_j, the highest wage that sector-j can offer without making losses, must be among the highest in all j.

Exercise: Show that $R(p, L)$ is not differentiable at $p = wa$ and that $x(p,L) >> 0$ implies that $p = wa$.

Consumers: They share the identical homothetic preferences, $E(p, U^h) = e(p)U^h$.

Budget Constraint: $e(p)U^h = wL^h$.

Individual Demand: $c^h = e_p(p)U^h = [e_p(p)/e(p)]wL^h$

Aggregate Demand: $c = \sum c^h = [e_p(p)/e(p)]w\sum L^h = [e_p(p)/e(p)]wL$.

Since e(p) is linear homogeneous, and $e_p(p)$, an N-dimensional column vector, is homogeneous of degree zero, aggregate demand can be rewritten to:

$$c(p, wL) = \frac{e_p(p)}{e(p)} wL = \frac{e_p(p/w)}{e(p/w)} L \,.$$

Furthermore, we assume that all the goods are *essential* in that consumption of each good is positive at any finite prices.

Autarky Equilibrium: Since all the goods are *essential*, $x^A = c^A >> 0$, which imply that

$$p^A = w^A a \qquad \& \qquad c^A = x^A = \frac{e_p(a)}{e(a)} L.$$

Note: The autarky relative prices are determined solely by the technology parameters, $a = (a_1, a_2, ..., a_N)$.

Two-Country World Economy: Home and Foreign, characterized by:

Home:	L	&	$a = (a_1, a_2, ..., a_N)$	&	e(p)
Foreign:	L*	&	$a^* = (a_1^*, a_2^*, ..., a_N^*)$	&	e*(p)

With little loss of generality, we assume that $A_j \equiv a_j^* / a_j$ is strictly decreasing in j. (This is mostly a matter of how to label goods.)

Characterizing Free Trade Equilibrium (in the absence of trade costs):

Step 1: The consumers everywhere purchase goods from the lowest cost producers: $p_j = Min\{wa_j, w^* a_j^*\}$. Suppose that good j is produced at Home and good k is produced at Foreign. Then, $A_j = a_j^*/a_j \geq \omega \equiv w/w^* \geq A_k = a_k^*/a_k$, hence $j \leq k$.

Step 2: We can immediate rule out the possibility $\omega > A_1$ or $\omega < A_N$. (How?)

Step 3: Suppose that there exists a m, such that $A_m > \omega > A_{m+1}$. Then, only Home produces j = 1, 2, …,m, and only Foreign produces j = m+1, m+2, …, N. Therefore,

$$p_j = \begin{cases} wa_j & \text{for j = 1, 2, …, m;} \\ w^* a_j^* & \text{for j = m+1, m+2, … , N,} \end{cases}$$

and

$$wL = w\sum_{j=1}^{m} L_j = \sum_{j=1}^{m}\left(\frac{p_j}{a_j}\right)L_j = \sum_{j=1}^{m} p_j X_j = \sum_{j=1}^{m} p_j\left[\left(\frac{e_j(p)}{e(p)}\right)wL + \left(\frac{e_j^*(p)}{e^*(p)}\right)w^* L^*\right].$$

Or

$$wL = wL\sum_{j=1}^{m}\alpha_j(p) + w^*L^*\sum_{j=1}^{m}\alpha_j^*(p),$$

where $\alpha_j(p) \equiv \dfrac{p_j e_j(p)}{e(p)}$ and $\alpha_j^*(p) \equiv \dfrac{p_j e_j^*(p)}{e^*(p)}$ are Good-j's share in the Home (Foreign) expenditures, which are homogeneous of degree zero in p.

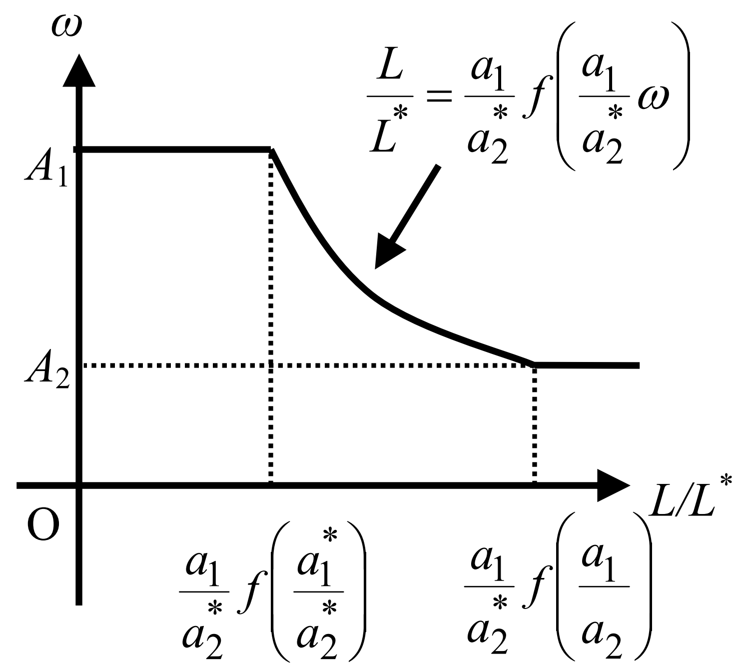
This can be further rewritten as the **Balanced Trade Condition (BT)**:

(BT): Home Imports = $\left[\sum_{j=m+1}^{N}\alpha_j(p)\right]wL = \left[\sum_{j=1}^{m}\alpha_j^*(p)\right]w^*L^*$ = Foreign Imports

For each m, we can solve (BT) for $\omega = w/w^*$. If the solution satisfies $A_m > \omega > A_{m+1}$, it is the equilibrium terms of trade.

Exercise: *How do we modify the argument to allow for* $\omega = A_m$*?*

Two-Sector Case: N = 2 and e(p) = e*(p):



If $A_1 > \omega > A_2$, m = 1 and $p_1 = wa_1;\ p_2 = w^* a_2^*$.

Hence, (BT) becomes

$$\frac{L}{L^*} = \frac{\alpha_1(p)}{\omega\alpha_2(p)} = \frac{p_1}{\omega p_2}\frac{c_1(p_1/p_2)}{c_2(p_1/p_2)} = \frac{a_1}{a_2^*} f\left(\frac{a_1}{a_2^*}\omega\right),$$

which is decreasing in ω, because the relative demand, c_1/c_2, is decreasing in the relative price. (This comes from e_{pp} being semi-negative definite.)

If $\frac{L}{L^*} < \frac{a_1}{a_2^*} f\left(\frac{a_1^*}{a_2^*}\right)$, $\omega = A_1$. Home produces only Good 1; Foreign produces both goods.

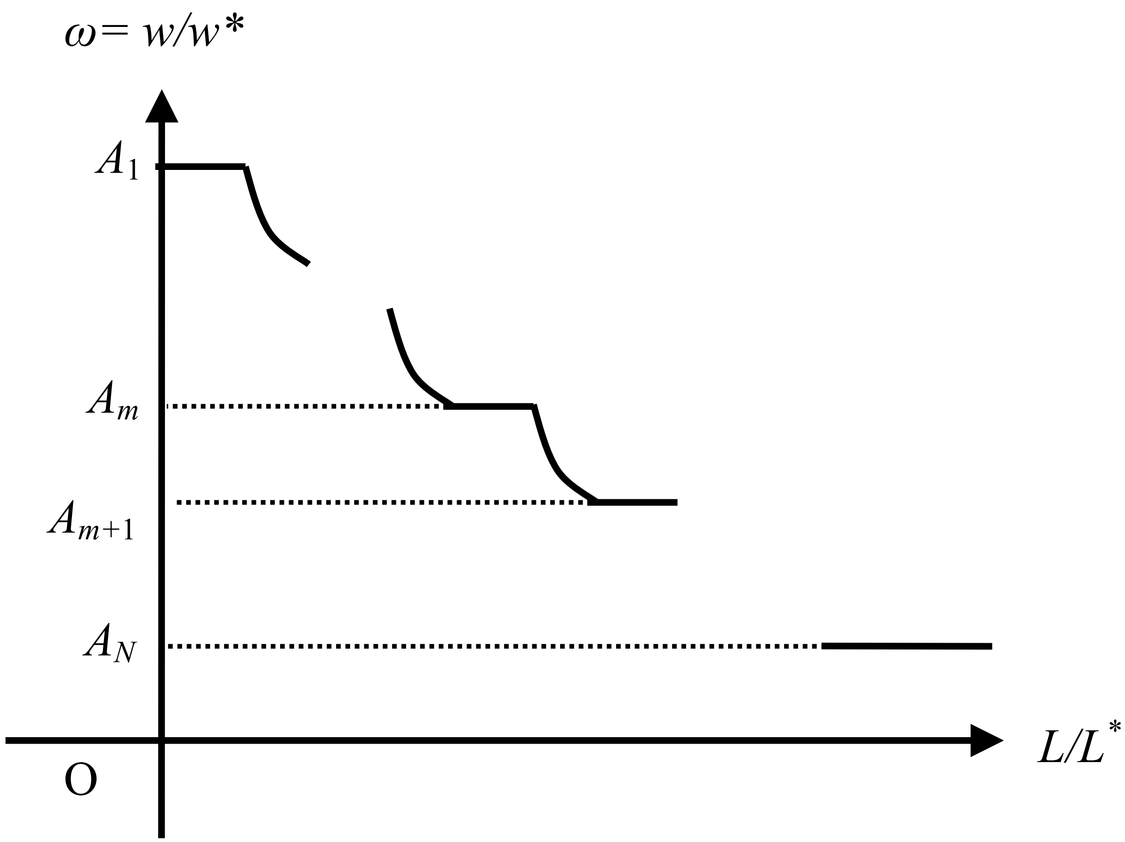
If $\frac{L}{L^*} > \frac{a_1}{a_2^*} f\left(\frac{a_1}{a_2}\right)$, $\omega = A_2$. Home produces both goods; Foreign produces Good 2 only.

Gains from Trade:

- Home gains from trade, iff $p \neq p^A = a$, i.e., iff $\omega > A_2$.
- Foreign gains from trade iff $p \neq p^{*A} = a^*$, i.e., iff $\omega < A_1$.

Cobb-Douglas Case: $\log U = \alpha_j \sum_{j=1}^{N} \log c_j$ and $\log U^* = \alpha_j^* \sum_{j=1}^{N} \log c_j^*$.

$$\frac{L}{L^*} = \frac{\alpha_1^* + \alpha_2^* + \ldots + \alpha_m^*}{(\alpha_{m+1} + \alpha_{m+2} + \ldots + \alpha_N)\omega} \qquad \text{for } A_m > \omega > A_{m+1}.$$



Welfare Measures (of a household endowed with one unit of labor):

$$\log U = \alpha_j \sum_{j=1}^{N} \log\left(\frac{\alpha_j w}{p_j}\right); \qquad \log U^* = \alpha_j^* \sum_{j=1}^{N} \log\left(\frac{\alpha_j^* w^*}{p_j}\right).$$

In Autarky, $$\log U^A = \alpha_j \sum_{j=1}^{N} \log\left(\frac{\alpha_j}{a_j}\right); \qquad \log U^{*A} = \alpha_j^* \sum_{j=1}^{N} \log\left(\frac{\alpha_j^*}{a_j^*}\right)$$

Gains from Trade:

$$\log\left(\frac{U}{U^A}\right) = \alpha_j \sum_{j=1}^{N} \log\left(\frac{wa_j}{p_j}\right) = \alpha_j \sum_{j=m+1}^{N} \log\left(\frac{\omega}{A_j}\right) > 0 \quad \text{if } \omega > A_N \text{, i.e., } p \neq p^A = a;$$

$$\log\left(\frac{U^*}{U^{*A}}\right) = \alpha_j^* \sum_{j=1}^{N} \log\left(\frac{w^* a_j^*}{p_j}\right) = \alpha_j^* \sum_{j=1}^{m} \log\left(\frac{A_j}{\omega}\right) > 0 \quad \text{if } \omega < A_1 \text{, i.e., } p \neq p^{*A} = a^*.$$

Thus,

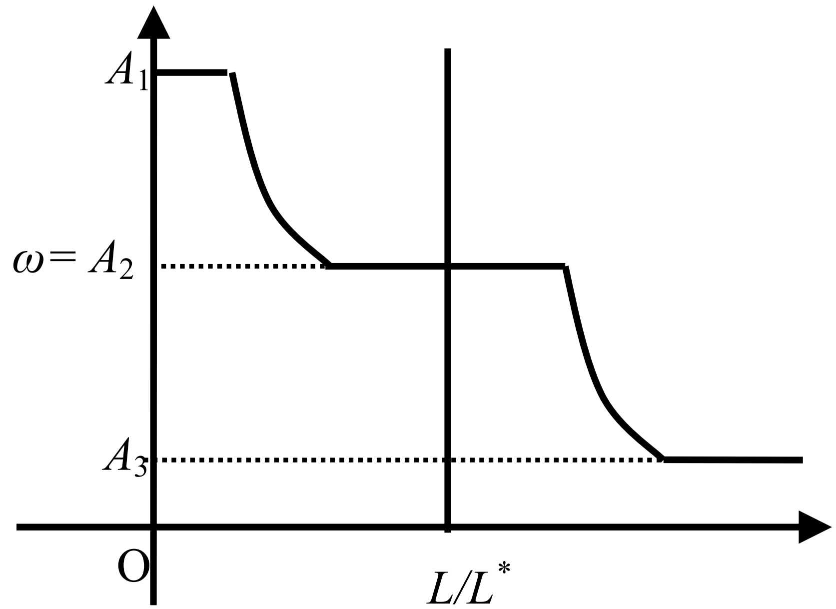
- At least one country gains from trade when $a \neq a^*$.
- Both countries gain from trade when $A_1 > \omega > A_N$.

Welfare Effects of Foreign Productivity Growth:

Suppose $N = 3$ and $A_1 > A_2 = \omega > A_3$. Choose the units of goods so that $a_j = 1$. (This is fine, because we change only the Foreign productivity.) Then, $a^*_j = A_j$ for all j, and

$$\log U = \log U^A + \alpha_3 \log\left(a_2^* / a_3^*\right),$$

where U^A is independent of Foreign productivity.



- Home welfare *improves* when Foreign productivity gains takes the form of a reduction in a^*_3.
- Home welfare *worsens* when Foreign productivity gains takes the form of a reduction in a^*_2.

Exercise: The discussion above assumes that $a^*_1 > a^*_2 = \omega > a^*_3$ both before and after the change. How should the discussion be modified if the decline in a^*_2 is large enough that $a^*_1 > \omega > a^*_2 > a^*_3$ holds after the change?

More generally, when $A_1 > A_m = \omega > A_N$,

$$\Delta \log U = \log\left(\frac{U}{U^A}\right) = \alpha_j \sum_{j=m+1}^{N} \log\left(\frac{\omega}{A_j}\right) = \alpha_j \sum_{j=m+1}^{N} \log\left(\frac{A_m}{A_j}\right)$$

- Home benefits from Foreign productivity gains in sectors m+1 to N, because it does not make Foreign labor more expensive, and hence makes Foreign imports cheaper.
- Home suffers from Foreign productivity gains in sector m, because it makes Foreign labor and hence Foreign imports produced in sectors m+1 to N more expensive.

Exercise: How do Foreign productivity gains affect the Foreign welfare? (Be careful! Foreign autarky welfare, U^{*A}, is not independent of a_j^*'s.)

A Ricardian Model with a Continuum of Goods: Dornbusch-Fischer-Samuelson (1977)

Tradeable Goods: $z \in [0,1]$.

Home: L, $a(z)$, $\log U = \int_0^1 \alpha(z) \log c(z) dz$

Foreign: L^*, $a^*(z)$, $\log U^* = \int_0^1 \alpha^*(z) \log c^*(z) dz$

where $A(z) \equiv a^*(z)/a(z)$ is strictly decreasing in z.

Patterns of Trade (PT) (in the absence of trade costs):

$$p(z) = \begin{cases} wa(z) < w^*a^*(z) & \text{for } z \in [0,m) \\ w^*a^*(z) < wa(z) & \text{for } z \in (m,1] \end{cases}$$

where m is the *marginal good* (and the *marginal sector*), satisfying

(PT) $\omega \equiv \dfrac{w}{w^*} = A(m)$,

which determines the Patterns of Trade (PT).

Balanced Trade (BT) Condition:

Since everyone buys the goods in [0, m) only from Home and the goods in (m,1] only from Foreign,

$$\text{Home Imports} = \left[\int_m^1 \alpha(z)dz\right] wL = \left[\int_0^m \alpha^*(z)dz\right] w^* L^* = \text{Foreign Imports}$$

or

(BT) $$\omega \equiv \frac{w}{w^*} = \frac{B^*(m)}{1-B(m)}\frac{L^*}{L},$$

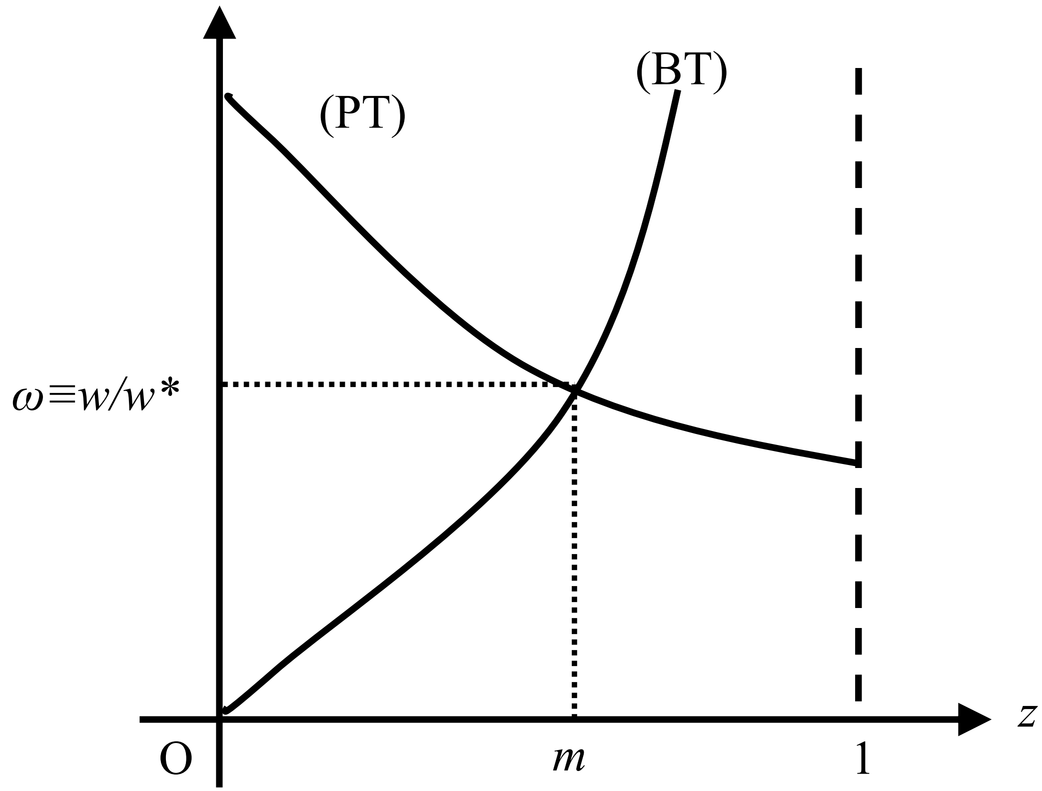
where $B(m) \equiv \int_0^m \alpha(z)dz$ $\left(B^*(m) \equiv \int_0^m \alpha^*(z)dz\right)$ is the share of the goods in [0, m] in the Home (Foreign) expenditure. Both increasing in m; $B(0) = B^*(0) = 0$; $B(1) = B^*(1) = 1$.

Joint Determination of the equilibrium terms of trade, ω, and equilibrium patterns of trade, m.

(PT) $\omega = A(m)$

and

(BT) $\omega = \dfrac{B^*(m)}{1 - B(m)} \dfrac{L^*}{L}.$



Welfare Measures: Indirect Utility of a Household endowed with one unit of labor:

$$\log U = \int_0^1 \alpha(z)\log\left(\frac{\alpha(z)w}{p(z)}\right)dz; \qquad \log U^* = \int_0^1 \alpha^*(z)\log\left(\frac{\alpha^*(z)w^*}{p(z)}\right)dz$$

In Autarky: $\log U^A = \int_0^1 \alpha(z)\log\left(\frac{\alpha(z)}{a(z)}\right)dz \qquad \log U^{*A} = \int_0^1 \alpha^*(z)\log\left(\frac{\alpha^*(z)}{a^*(z)}\right)dz$

Gains from Trade:

$$\log\left(\frac{U}{U^A}\right) = \int_0^1 \alpha(z)\log\left(\frac{wa(z)}{p(z)}\right)dz = \int_m^1 \alpha(z)\log\left(\frac{A(m)}{A(z)}\right)dz > 0.$$

$$\log\left(\frac{U^*}{U^{*A}}\right) = \int_0^1 \alpha^*(z)\log\left(\frac{w^*a^*(z)}{p(z)}\right)dz = \int_0^m \alpha^*(z)\log\left(\frac{A(z)}{A(m)}\right)dz > 0.$$

Some Comparative Statics:

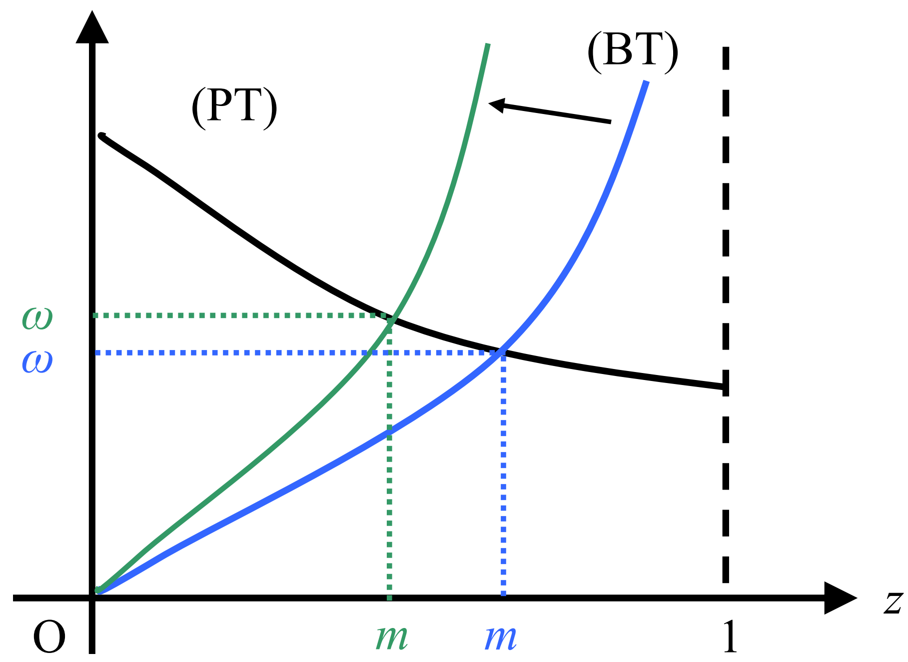
***Relative Country (Population) Size*:**

$L^*/L\uparrow \rightarrow\ \omega\uparrow$ and $m\downarrow$

$$\partial \log U = -[1-B(m)]\xi(m)\partial m > 0\,;$$

$$\partial \log U^* = B^*(m)\xi(m)\partial m < 0$$

where $\xi(z) \equiv -\dfrac{d\log A(z)}{dz} = -\dfrac{A'(z)}{A(z)} > 0$.

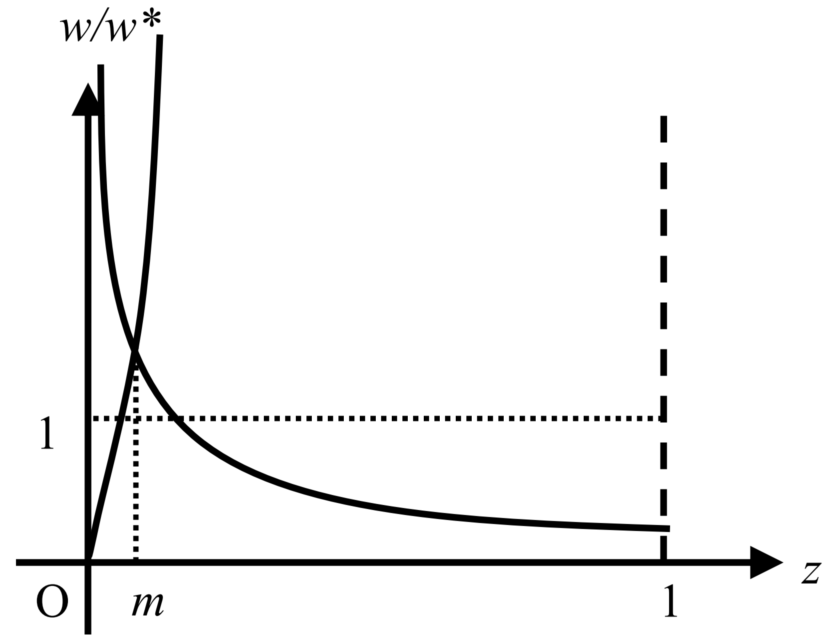


Some Interpretations:

- Faster population growth in the South (Foreign) than in the North (Home) makes Southern labor cheaper. The North gains not only because their import prices go down, but also because they can further specialize into activities that they are particularly good at. (Of course, this also means that, due to cheap labor in the South, North loses its "competitiveness" in certain sectors, which immigrate to the South.)

- South consists of many similar countries, some of which are previously not integrated into the global economy. This experiment may be interpreted as looking at the effects on the North when more and more countries in the South joining the global economy.

- Smaller countries enjoy higher per capita income. Small countries need not be good in many sectors to stay rich. As long as there is enough world demand for the few things they are good at doing, they can maintain the high wage income. This may explain why countries like Norway and Switzerland are rich in spite that their climate and geography are not suitable for most economic activities.

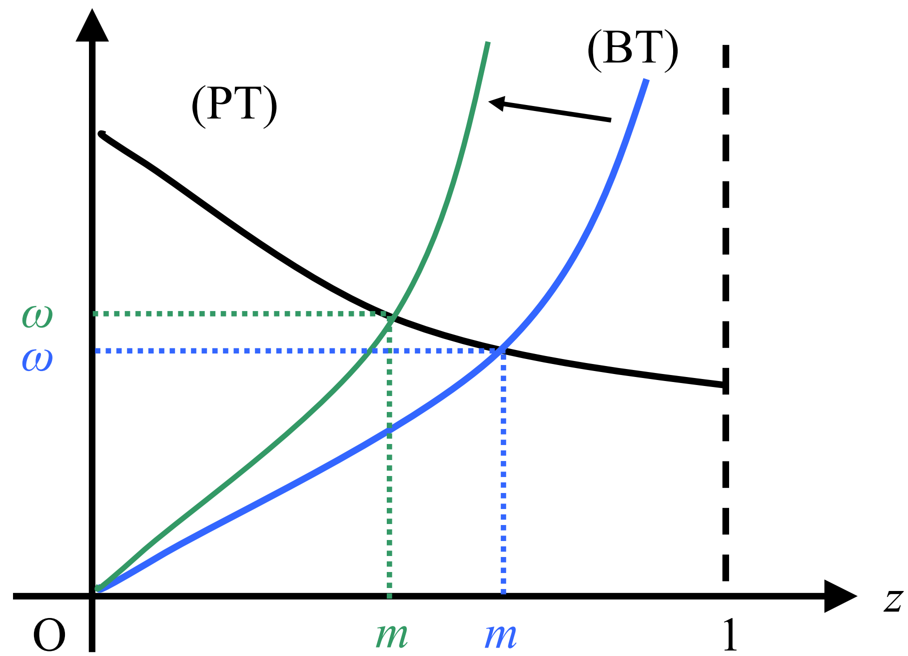


Broad Implications

In spite of the CRS technologies, the *endogeneity of the terms of trade* introduces *de facto* diminishing returns. ***Some Cautions for GDP and Growth Accountings!***

Foreign Taste Shifts Towards Home Goods: $B^*(z)\uparrow$ for each z → $\omega\uparrow$ and $m\downarrow$

$$\partial \log U = -[1 - B(m)]\xi(m)\partial m > 0$$



(Infinitesimal) Technology Changes: $g(z) \equiv -\dfrac{\partial a(z)}{a(z)}$, $g^*(z) \equiv -\dfrac{\partial a^*(z)}{a^*(z)}$.

(Why infinitesimal? We want to avoid the need for relabeling the goods.)

Let $\alpha(z) = \alpha^*(z) = 1$ and hence, $B(z) = B^*(z) = z$, to simplify the notation. Then,

$$A(m) = \omega \equiv \frac{w}{w^*} = \frac{m}{1-m}\frac{L^*}{L}$$

$$\log U = \int_0^1 \log\left(\frac{w}{p(z)}\right)dz = -\int_0^m \log a(z)dz + \int_m^1 \log\left(\frac{w}{a^*(z)w^*}\right)dz.$$

Total differentiation yields,

$$\frac{dU}{U} = \int_0^m g(z)dz + \int_m^1 g^*(z)dz + \left[\frac{1-m}{1+\xi(m)m(1-m)}\right][g(m) - g^*(m)],$$

which breaks up the effect into three components:

- Direct effect of Home productivity gains in the Home active sectors (1st term)
- Direct effect of Foreign productivity gains in the Foreign active sectors (2nd term)
- Indirect *Terms of Trade* effect (3rd term)

Uniform Global Productivity Gains: $g(z) = g^*(z) = g > 0$.

$$d\log U = g > 0.$$

Uniform Foreign Productivity Gains: $g(z) = 0$; $g^*(z) = g^* > 0$.

$$d\log U = \left[\frac{m(1-m)^2\xi(m)}{1+\xi(m)m(1-m)}\right]g^* > 0.$$

Note: From Home's point of view, this is isomorphic to the Foreign population growth.

Uniform Home Productivity Gains: $g(z) = g > 0$; $g^*(z) = 0$.

$$0 < d\log U = \left[\frac{1+\xi(m)m^2(1-m)}{1+\xi(m)m(1-m)}\right]g < g$$

Home's welfare gains are less than 100% of its productivity gains, because some of the gains spill over to Foreign.

Note: These results on *uniform* productivity gains do not carry over to more general preferences. However, they offer a useful benchmark for evaluating the effects of *biased* productivity gains.

***Biased Foreign Productivity Gains*:** $g(z) = 0$ and $g^*(z) > 0$.

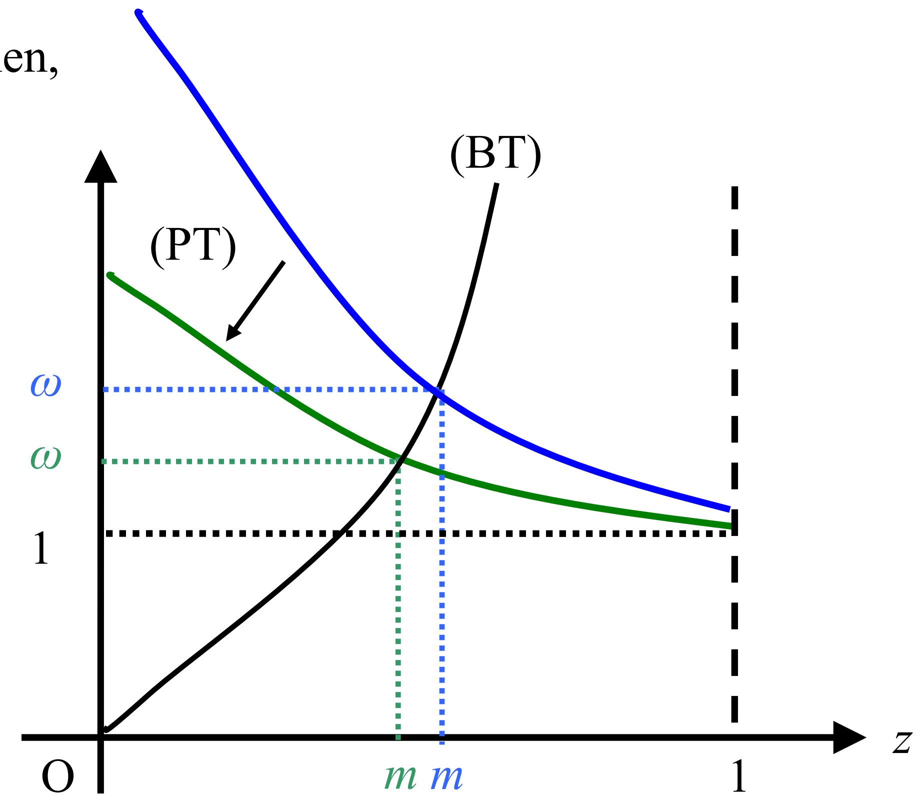
Home may suffer if they are larger near the marginal sector.

Example* 1*: Let $a(z) = 1$; $a^*(z) = \Lambda^{1-z}$, $\Lambda > 1$. Then,

$$\log U = \frac{(1-m)^2}{2}\log\Lambda;$$

$$\log U^* = -\frac{1-m^2}{2}\log\Lambda,$$

$$\Lambda^{1-m} = \frac{m}{1-m}\frac{L^*}{L}.$$



A decline in Λ causes faster productivity gains in lower-indexed sectors: $g^*(z) = (1-z)\partial\log(\Lambda)$.

A decline in Λ (Foreign productivity gains) could lead to a lower U.

Interpretation: Technology Catching Up

Λ represents the extent to which Foreign *lags behind* Home technologically. (See my Palgrave entry for a story behind it.) Home has absolute advantage in all sectors, but Foreign has comparative advantage in the high-indexed sectors. As Foreign *narrows the gap*, it becomes more similar to Home, and Home gains little from trading with a country similar to itself. If Foreign *catches up* completely, $\Lambda = 1$, Home loses all the gains from trade as the two countries become identical.

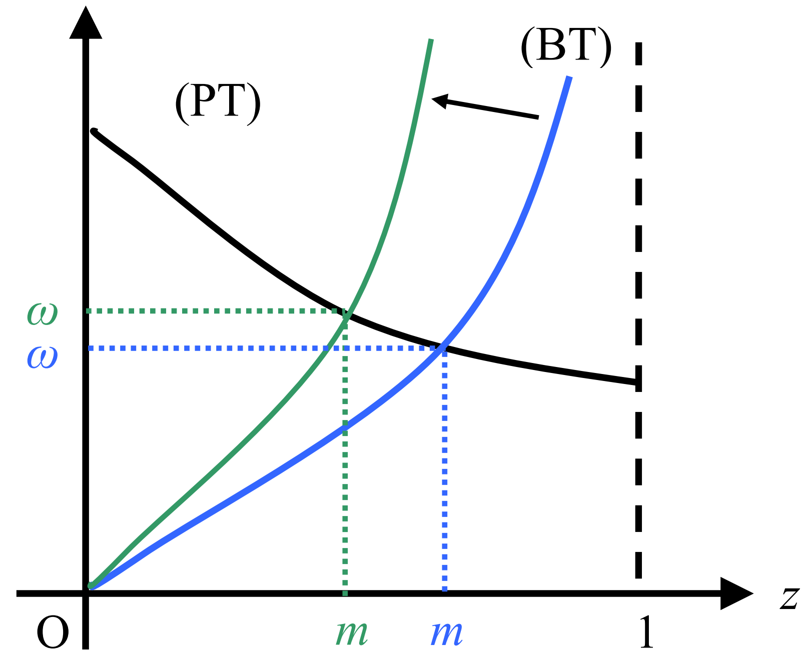
Example 2: $A(z) = [A_0(z)]^{\lambda}$, where $A_0(z)$ is strictly decreasing in z and $A_0(z) > 1$ and $\lambda > 0$. (See Krugman (1986) for a story behind it.) Letting $\lambda \rightarrow 0$ has similar effects. As in Example 1, it is crucial that the rate of productivity gains is larger around the marginal sector than in the higher-indexed sectors, which makes the Foreign import prices go up when measured in Home labor.

Note: We will later look at the Eaton-Kortum extension of the DFS model. The parameterization of the technologies used by EK does not allow for this kind of biased technical change.

Transfer with Taste Differences: A lump-sum transfer, w^*T, from Foreign to Home.

Home Expenditure: $E = wL + w^*T$;
Foreign Expenditure: $E^* = w^*L^* - w^*T$.
Home GDP Accounting: $wL = B(m)E + B^*(m)E^*$
→

(BT) $$\omega \equiv \frac{w}{w^*} = \frac{B^*(m)L^* + \{B(m) - B^*(m)\}T}{\{1 - B(m)\}L}.$$



If $B(z) > B^*(z)$ for all z (home-biased in tastes),

$T\uparrow$ → $\omega\uparrow$ and $m\downarrow$.

The Donor (Recipient) suffers (gains) not only from the direct effect of the transfer, but also from the indirect effect of the terms of trade change caused by the transfer.

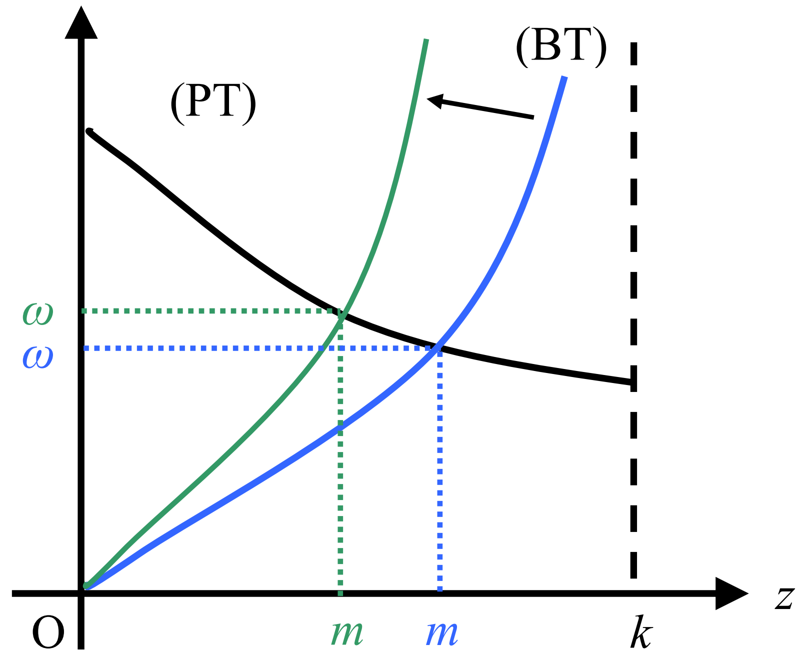
Note: With the transfer, Trade Account is not balanced, but the overall BOP Account is, as Home GDP Accounting implies;

$$\textit{Home Transfer Surplus} = w^*T = (1 - B(m))E - B^*(m)E^* = \textit{Home's Trade Deficit}$$

Nontradeables, Trade Costs and Globalization

Transfer with Nontradeables:

- Home and Foreign share the symmetric Cobb-Douglas preferences, so that $B(z) = B^*(z) = z$. $z \in [0,k)$ are all tradeables at zero cost, while $z \in$ [k, 1] are nontradeable. Thus, 100(1 − k)% of the expenditure goes to the nontradeables.
- $A(z)$ is strictly decreasing in $z \in [0,k)$.



→ $\quad wL = m(E + E^*) + (1-k)E$,

where $E = wL + w^*T$ and $E^* = w^*L^* - w^*T$.

(BT) $\qquad \omega \equiv \dfrac{w}{w^*} = \dfrac{mL^* + (1-k)T}{(k-m)L}$.

Hence, $T\uparrow$ → $\omega\uparrow$ and $m\downarrow$.

Home biased in demand arises naturally from the presence of nontradables.

Note: Again, *Home Transfer Surplus* = $w^*T = (k-m)E - mE^*$ = *Home's Trade Deficit.*

Uniform vs. Non-Uniform Globalization: Matsuyama (2007)

Question: What are the welfare effects of globalization that turns some nontradeables into tradeables?

Setup: L ($L*$) households at Home (Foreign). Each household supplies one unit of labor and shares the symmetric Cobb-Douglas preferences defined over $z \in [0,1]$.

$$\text{Max } \log U = \int_0^1 \log[c(z)]dz, \text{ s.t. } \int_0^1 p(z)c(z)dz \le w \rightarrow \log U = \int_0^1 \log[w/p(z)]dz$$

$$\text{Max } \log U^* = \int_0^1 \log[c^*(z)]dz, \text{ s.t. } \int_0^1 p^*(z)c^*(z)dz \le w^* \rightarrow \log U^* = \int_0^1 \log[w^*/p^*(z)]dz$$

Unit Labor Requirement; $a(z)$, $a^*(z)$. Define $A(z) \equiv a^*(z)/a(z)$.

$G(A)$: the measure of the tradeable goods with $A(z) \le A$.

$H(A)$: the measure of the nontradeable goods with $A(z) \le A$.

$F(A) \equiv G(A) + H(A)$; the measure of the goods with $A(z) \le A$.

Autarky Equilibriums: $p(z) = a(z)w$, $p^*(z) = a^*(z)w^*$

$$\rightarrow \log U^A = -\int_0^1 \log[a(z)]dz; \quad \log U^{*A} = -\int_0^1 \log[a^*(z)]dz$$

Trade Equilibrium: If z is tradeable <u>and</u> $a(z)w > a^*(z)w^*$, or $\omega \equiv w/w^* > a^*(z)/a(z) \equiv A(z)$, $p(z) = a^*(z)w^*$. Otherwise, $p(z) = a(z)w$.

$$\rightarrow \Delta\log U \equiv \log\left(\frac{U}{U^A}\right) = \int_0^1 \log\left(\frac{a(z)w}{p(z)}\right)dz = \int_0^{\omega} \log[\omega/A]dG(A) > 0.$$

Likewise, $$\Delta\log U^* \equiv \log\left(\frac{U^*}{U^{*A}}\right) = \int_{\omega}^{\infty} \log[A/\omega]dG(A) > 0.$$

Since Home imports all tradeable goods whose $A(z) < \omega$ and Foreign imports all the tradeable goods $A(z) > \omega$, the balanced trade implies $G(\omega)wL = [G(\infty) - G(\omega)]w^*L^*$, or

(BT): $$\frac{G(\infty) - G\_(\omega)}{G\_(\omega)} \geq \frac{\omega L}{L^*} \geq \frac{G(\infty) - G(\omega)}{G(\omega)},$$ if G is allowed to have mass points.

(BT) determines the equilibrium terms of trade, ω.

Uniform Globalization: $G(A) = \gamma F(A)$, and $H(A) = (1-\gamma)\mathrm{F}(A)$.

(BT): $$\frac{G(\infty)-G_{-}(\omega)}{G_{-}(\omega)} \geq \frac{\omega L}{L^{*}} \geq \frac{G(\infty)-G(\omega)}{G(\omega)}$$

is independent of γ, and hence, so is ω. Therefore,

$$\Delta \log U = \gamma \int_{0}^{\omega} \log[\omega/A] dF(A); \qquad \Delta \log U^{*} = \gamma \int_{\omega}^{\infty} \log[A/\omega] dF(A)$$

are both increasing in γ.

In this case, the newly tradeables do not change the patterns of comparative advantage, and hence the globalization does not affect TOT.

What if globalization is non-uniform?

Non-Uniform Globalization: A Two-Sector Example

Unit labor requirement takes only two values; $A_1 = a^*_1/a_1 > a^*_2/a_2 = A_2$. Thus, Home (Foreign) has comparative advantage in Sector 1 (Sector 2).

- $A(z) = A_1$ for α_1 fraction of the goods, of which γ_1 fraction is tradeable.
- $A(z) = A_2$ for $\alpha_2 = 1-\alpha_1$ fraction of the goods, of which γ_2 fraction is tradeable.

If $A_1 > \omega > A_2$, Home exports $100\gamma_1$% of Good 1; Foreign exports $100\gamma_2$% of Good 2.

$$\rightarrow \quad \gamma_2\alpha_2 wL = \gamma_1\alpha_1 w^* L^* \rightarrow A_1 > \omega = \frac{\gamma_1\alpha_1}{\gamma_2\alpha_2}\frac{L^*}{L} > A_2$$

$$\rightarrow \quad \Delta\log U = \gamma_2\alpha_2 \log\left(\frac{\omega}{A_2}\right) = \gamma_2\alpha_2 \log\Gamma\left(\frac{\gamma_1}{\gamma_2}\right) > 0, \text{ where } \Gamma \equiv \frac{1}{A_2}\frac{\alpha_1}{\alpha_2}\frac{L^*}{L}.$$

$$\Delta\log U^* = \gamma_1\alpha_1 \log\left(\frac{A_1}{\omega}\right) = \gamma_1\alpha_1 \log\Gamma^*\left(\frac{\gamma_2}{\gamma_1}\right) > 0, \text{ where } \Gamma^* \equiv A_1\frac{\alpha_2}{\alpha_1}\frac{L}{L^*}.$$

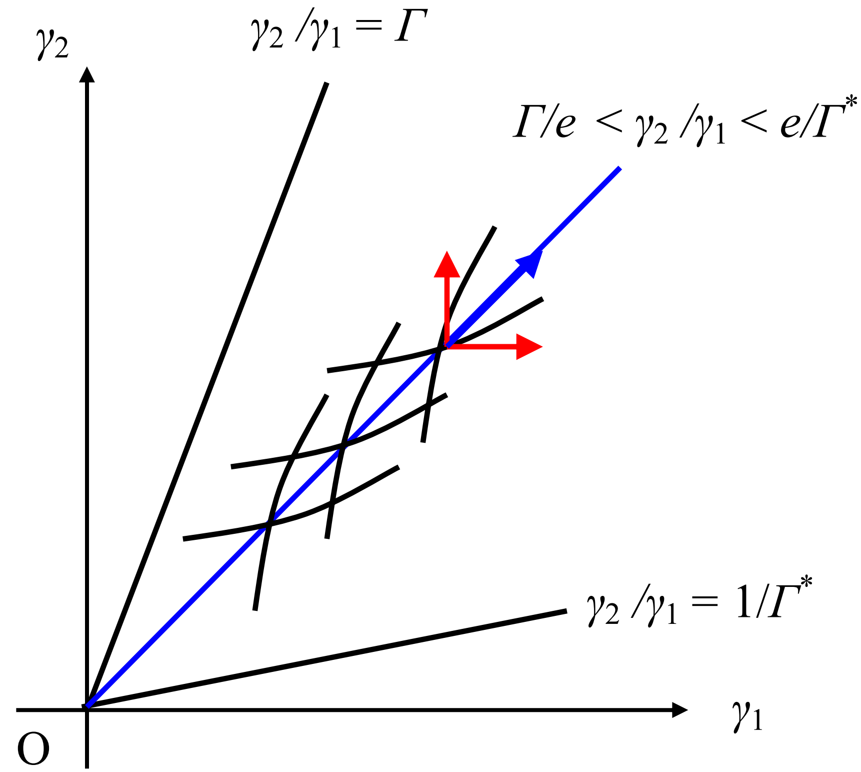
If $\frac{\gamma_1\alpha_1}{\gamma_2\alpha_2}\frac{L^*}{L} \geq A_1 \rightarrow A_1 = \omega > A_2 \rightarrow \Delta\log U = \gamma_2\alpha_2 \log\left(\frac{A_1}{A_2}\right) > 0 \;\&\; \Delta\log U^* = 0.$

If $\frac{\gamma_1\alpha_1}{\gamma_2\alpha_2}\frac{L^*}{L} \leq A_2 \rightarrow A_2 = \omega < A_1 \rightarrow \Delta\log U = 0 \;\&\; \Delta\log U^* = \gamma_1\alpha_1 \log\left(\frac{A_1}{A_2}\right) > 0.$

Let $1/\Gamma^* < \gamma_2/\gamma_1 < \Gamma$, so that $\omega = \dfrac{\gamma_1\alpha_1}{\gamma_2\alpha_2}\dfrac{L^*}{L}$ and

$$\Delta \log U(\gamma_1, \gamma_2) \equiv \alpha_2\gamma_2 \log \Gamma\left(\frac{\gamma_1}{\gamma_2}\right) > 0;\quad \Delta \log U^*(\gamma_1, \gamma_2) \equiv \alpha_1\gamma_1 \log \Gamma^*\left(\frac{\gamma_2}{\gamma_1}\right) > 0.$$

- Home always gains from a globalization in Sector 1 (i.e., a higher γ_1).
- Home *loses* from a globalization in Sector 2 (i.e., a higher γ_2), if $\mathrm{Max}\{1/\Gamma^*, \Gamma/e\} < \gamma_2/\gamma_1 < \Gamma$.
- Foreign always gains from a globalization in Sector 2 (a higher γ_2).
- Foreign *loses* from a globalization in Sector 1 (a higher γ_1), if $1/\Gamma^* < \gamma_2/\gamma_1 < \mathrm{Min}\{\Gamma, e/\Gamma^*\}$.



Intuition:
TOT may deteriorate more than enough to offset the benefits of more trade opportunities.

Non-Uniform Globalization: A Continuum Case

- $z \in [0, k)$ are all originally tradeables, for which $A(z)$ is strictly decreasing, so that, given $A(m) = w/w^*$, Home produces all $z \in [0, m)$ and Foreign produces all $z \in [m, k)$.
- $A(z) = A$ for all nontradeables, $z \in [k, 1]$, but a fraction γ of these goods of these goods become newly tradeable at zero cost.

If $w/w^* > A$, all of the newly tradeables are produced at Foreign. Because Home produces all the originally tradable goods in $[0, m]$ for both countries and $(1-\gamma)(1-k)$ fraction of the goods (those which remain nontradeable) locally,

$$wL = m(wL+w^*L^*) + (1-\gamma)(1-k)wL \qquad \leftrightarrow \qquad \frac{w}{w^*} = \frac{m}{k+\gamma(1-k)-m}\left[\frac{L^*}{L}\right]$$

If $w/w^* < A$, all of the newly tradeables are produced at Home. Because Home produces $m+\gamma(1-k)$ fraction of the goods for both countries and $(1-\gamma)(1-k)$ fraction of the goods locally,

$$wL = [m+\gamma(1-k)](wL+w^*L^*)+(1-\gamma)(1-k)wL \qquad \leftrightarrow \qquad \frac{w}{w^*} = \frac{m+\gamma(1-k)}{k-m}\left[\frac{L^*}{L}\right].$$

Otherwise, $w/w^* = A$.

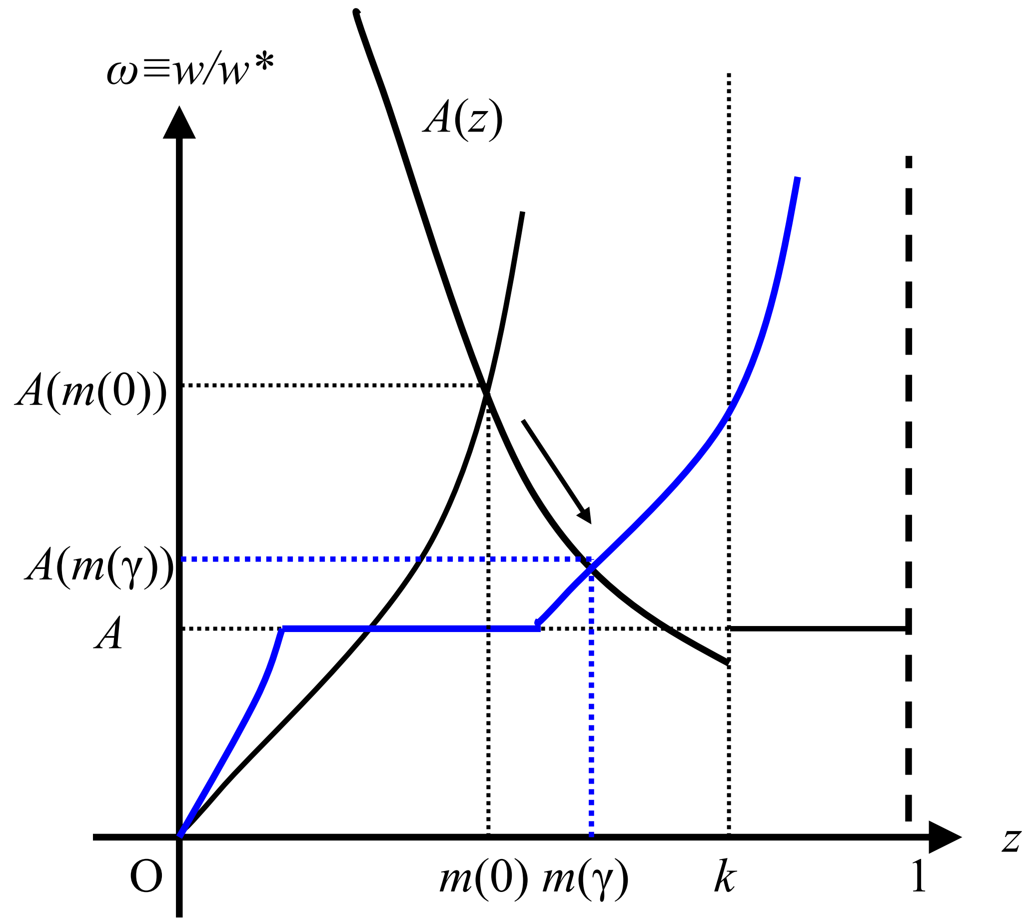
A higher γ shifts the BT to the right above $w/w^* = A$ and to the left below $w/w^* = A$.

Suppose that, before globalization, $\gamma = 0$, $\omega = A(m(0)) > A$.

The arrow indicates the shift caused by an increase in γ.

When some nontraded sectors are opened up, Home stops producing the new tradeables and starts producing and exporting the goods in $(m(0), m(\gamma)]$, which it previously imported.

ω declines from $A(m(0))$ to $A(m(\gamma))$.



Home & Foreign Welfares:

$$\log U(\gamma) = \int_{m(\gamma)}^{k} \log\left[\frac{A(m(\gamma))}{A(z)}\right] dz + \gamma(1-k)\log\left[\frac{A(m(\gamma))}{A}\right];$$

$$\log U^*(\gamma) = m(\gamma)\log\left[\frac{A}{A(m(\gamma))}\right] + \int_{m(\gamma)}^{k} \log\frac{A}{A(z)} dz - \log A,$$

with the normalization, $A(z) = a^*(z)/a(z) = a^*(z)$ for all $z \in [0,1]$.

A globalization (a higher γ) affects the Home welfare through ***Two Effects***:

- **Positive Reallocation Effect:** Home labor moves to the sectors where they have higher relative efficiency, that is, from A to $A(m(\gamma))$ or higher.
- **Negative Terms of Trade Effect:** $\omega = A(m(\gamma))$, deteriorates.

The overall effect is generally ambiguous. However, if a higher γ brings down $\omega = A(m(\gamma))$ sufficiently close to A, the positive reallocation effect is dominated by the negative terms of trade effect, so that a further globalization harms the Home welfare.

Foreign always benefits from this type of globalization, as both effects operate positively.

Costly Trade: Iceberg Formulation

- Instead of dividing the goods into the two categories, the nontradeable and the tradeable with zero cost, let all the goods, $z \in [0,1]$, be tradeable at *some* costs.
- When shipped abroad, they are lost in transit (hence called Iceberg) and only a fraction $\gamma < 1$ arrives to the destination. This means that, in order to supply one unit of each good, the exporter must produce $1/\gamma > 1$ units of the good.

Goods Prices & Patterns of Trade:

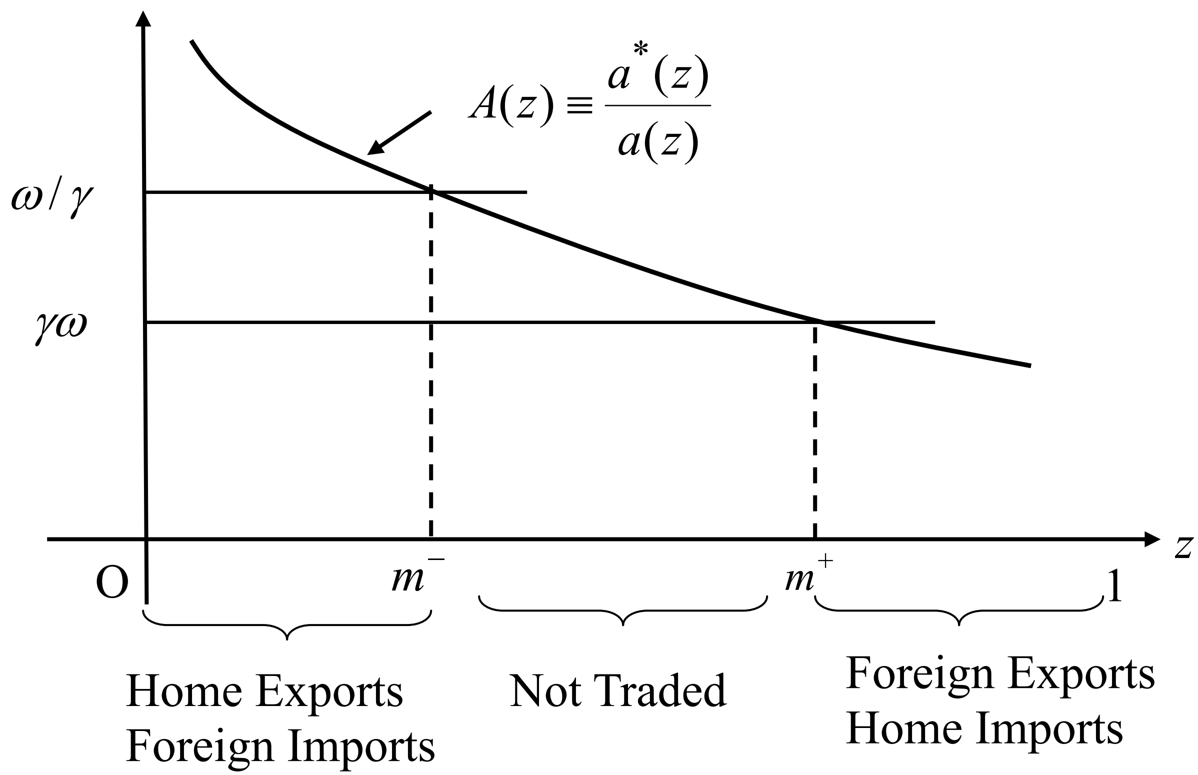
$$p(z) = \mathrm{Min}\ \{wa(z), w^*a^*(z)/\gamma\} = \begin{cases} wa(z) < w^*a^*(z)/\gamma & \text{for } z \in [0,m^+) \\ w^*a^*(z)/\gamma < wa(z) & \text{for } z \in (m^+,1] \end{cases}$$

$$p^*(z) = \mathrm{Min}\ \{wa(z)/\gamma, w^*a^*(z)\} = \begin{cases} wa(z)/\gamma < w^*a^*(z) & \text{for } z \in [0, m^-) \\ w^*a^*(z) < wa(z)/\gamma & \text{for } z \in (m^-,1] \end{cases}$$

where $m^- < m^+$ are the *marginal goods* (and the *marginal sectors*), determined by

(PT): $$\frac{w}{\gamma w^*} = A(m^-) > A(m^+) = \gamma \frac{w}{w^*}$$

- Home produces all the goods in $[0, m^-)$ for both countries.
- Foreign produces all the goods in $(m^+, 1]$ for both countries.
- Each country produces the goods in $[m^-, m^+]$, which are (endogenously) nontraded.
- Traded goods prices are not equalized across countries.



Balanced Trade (BT) Condition:

$$\text{Home Imports} = [1 - B(m^+)]wL = B^*(m^-)w^*L^* = \text{Foreign Imports}$$

→

(BT) $$\omega \equiv \frac{w}{w^*} = \frac{B^*(m^-)}{1 - B(m^+)}\left[\frac{L^*}{L}\right].$$

The three endogenous variables, m^-, m^+, and w/w^* are determined jointly by (BT) and the two equations in (PT).

One may proceed to analyze the effects of globalization by increasing γ. But, it is difficult to obtain unambiguous results without further assumptions.

Exercises:

- Find an example where a higher γ leads to a decline in m^- (or an increase in m^+).
- Find an example where a higher γ causes the welfare of one country to decline.

Tradeable-Nontradeable (TN) Dichotomy vs. Iceberg Costs

- Most goods can be traded at some finite costs. This makes *Iceberg* more appealing.
- Goods differ widely in trade costs. This makes the *TN Dichotomy* more appealing.
- For most purpose, I find TN dichotomy more tractable.
- However, there are some issues that cannot be addressed by TN dichotomy.

Itoh-Kiyono (1987): Trade Policies in Ricardian models with many goods.

Using a three-sector Ricardian model as well as the DFS model, they showed that the standard results on trade policies in the two-sector model, such as

- The equivalence of the export and import taxes (Lerner 1936):
- The export subsidies always reduce the national welfare:

are of very limited value in a world with many tradeable goods, as these results are applied only for the taxes and subsidies imposed *uniformly* across all export (or import) sectors.

For example, the selective subsidies targeted to the sectors near the marginal sectors may be able to improve the nation's welfare (at the expense of the ROW).

Non-Homothetic Preferences: Structural Change and North-South Trade

So far, we have assumed that preferences are homothetic (often Cobb-Douglas).

- Homotheticity implies that the rich & the poor consume goods in the same proportions.
- With Cobb-Douglas, each sector accounts for a fixed share of the total expenditure.

Empirically, they are clearly false. Conceptually, too restrictive for thinking about many important issues related to growth and development.

- Engel's Law
- US, EU, and Japan are the three biggest markets for SUV; China, India, and Indonesia are the three biggest markets for motorbikes.
- Fisher-Clark-Kuznets thesis; as economies develop, sectoral compositions change; The decline of agriculture, the rise and fall of manufacturing, and the rise of service sectors.
- Prebisch-Singer thesis; the long run trend that TOT moves in favor of the rich North and against the poor South.

We now look at various attempts to address some of these issues within the Ricardian framework by incorporating non-homothetic or non-Cobb-Douglas preferences.

Matsuyama (2009): Structure Change in an Interdependent World

Two Countries: Home and Foreign (*) with labor endowment normalized as $L = L^* = 1$.
Home (Foreign) wage: w (w^*).

Three Goods:

Numeraire (*O*); tradeable at zero cost;
No production. Endowment of y units
Manufacturing (*M*); tradeable at zero cost;
Home (Foreign) unit labor requirement in *M*; a_M (a_M*).
Services (*S*): nontradeable;
Home (Foreign) unit labor requirement in *S*: a_S (a_S*).

Home price of S: $p_S = a_S w$

Foreign price of *S*: $p_S^* = a_S^* w^*$

World Price of *M*: $p_M = a_M w = a_M^* w^*$

whenever both countries produce both *M* and *S*.

Home Households: *Stone-Geary* Preferences

$$U = \begin{cases} (c_O)^{\alpha}\left[\beta_M(c_M-\gamma)^{\theta}+\beta_S(c_S)^{\theta}\right]^{\frac{1-\alpha}{\theta}} & \text{for } \theta < 1,\ \theta \neq 0, \\ (c_O)^{\alpha}(c_M-\gamma)^{\beta_M(1-\alpha)}(c_S)^{\beta_S(1-\alpha)} & \text{for } \theta = 0. \end{cases}$$

If $\gamma > 0$, the income elasticity of demand for M is less than one.
If $\theta < 0$, the price elasticity of relative demand of M & S, $\sigma \equiv 1/(1-\theta)$, is less than one.

Home Budget Constraint: $c_O + p_M c_M + p_S c_S \leq y + w$

Home Demand Schedules for *O* and *S*:

$$c_O = \alpha(y+w-\gamma p_M), \qquad c_S = \frac{(\beta_S)^{\sigma}(p_S)^{-\sigma}(1-\alpha)(y+w-\gamma p_M)}{(\beta_M)^{\sigma}(p_M)^{1-\sigma}+(\beta_S)^{\sigma}(p_S)^{1-\sigma}}.$$

Likewise,
Foreign Demand Schedules for *O* and *S*:

$$c_O^* = \alpha(y+w^*-\gamma p_M), \qquad c_S^* = \frac{(\beta_S)^{\sigma}(p_S^*)^{-\sigma}(1-\alpha)(y+w^*-\gamma p_M)}{(\beta_M)^{\sigma}(p_M)^{1-\sigma}+(\beta_S)^{\sigma}(p_S^*)^{1-\sigma}}.$$

Market Clearing Conditions:

$$c_O + c_O^* = 2y; \quad a_S c_S = 1 - L_M; \qquad a_s^* c_S^* = 1 - L_M^*,$$

where L_M (L_M^*) is Home (Foreign) Manufacturing Employment Share.

Equilibrium Employment Shares:

$$L_M = \frac{\dfrac{\alpha}{2}\left(1 - \dfrac{a_M}{a_M^*}\right) + \gamma a_M + \left(\dfrac{\beta_M}{\beta_S}\right)^{\sigma}\left(\dfrac{a_M}{a_S}\right)^{1-\sigma}}{1 + \left(\dfrac{\beta_M}{\beta_S}\right)^{\sigma}\left(\dfrac{a_M}{a_S}\right)^{1-\sigma}};$$

$$L_M^* = \frac{\dfrac{\alpha}{2}\left(1 - \dfrac{a_M^*}{a_M}\right) + \gamma a_M^* + \left(\dfrac{\beta_M}{\beta_S}\right)^{\sigma}\left(\dfrac{a_M^*}{a_S^*}\right)^{1-\sigma}}{1 + \left(\dfrac{\beta_M}{\beta_S}\right)^{\sigma}\left(\dfrac{a_M^*}{a_S^*}\right)^{1-\sigma}}.$$

Suppose either $\gamma > 0$ & $\sigma = 1$, or $\gamma = 0$ & $\sigma < 1$. Then,

Global Productivity Growth in M: $\frac{\Delta a_M}{a_M} = \frac{\Delta a_M^*}{a_M^*} < 0$ → $\Delta L_M < 0$; $\Delta L_M^* < 0$.

National Productivity Growth in M: $\frac{\Delta a_M}{a_M} < 0 = \frac{\Delta a_M^*}{a_M^*}$ → ΔL_M ?? 0; $\Delta L_M^* < 0$.

- The model suggests **a global trend of manufacturing decline due to productivity growth in manufacturing.**
- *However,* it does *not* suggest that faster productivity growth in a country would lead to faster decline in *its* manufacturing sector.
- **In *cross-sections of countries*, manufacturing productivity might be *positively* correlated with the manufacturing employment share, due to *comparative advantage.***

e.g. Higher productivity growth in the German or Japanese manufacturing sector means that the manufacturing sectors must decline *somewhere* in the world, but *not necessarily* in Germany or Japan.

Message: Imagine:

- An economist wants to test the hypothesis that productivity growth in manufacturing causes a decline in the manufacturing employment.
- He develops a *closed* economy model.
- He runs cross-country regressions under the *false* maintained hypothesis that each country is *in autarky*.

Then, he would find the evidence that reject the hypothesis *convincingly,* even though the hypothesis *is* correct.

Exercise: **Balassa-Samuelson Effect**

In this model, where Nontraded S-Sector compete with Traded M-sector for labor, the relative price of S satisfies:

$$\frac{p_S}{p_S^*} = \frac{a_S}{a_S^*}\frac{w}{w^*} = \frac{a_S}{a_S^*}\frac{a_M^*}{a_M}.$$

This is useful for understanding why many services (haircut, restaurant food, etc.) are much cheaper in developing countries. Is it because developing countries are much more efficient in the service sector? Or, is it because developing countries are much more inefficient the tradeable sector?

Key Features of Stone-Geary preferences:

- *Average* propensity to consume differs across goods and changes with income, hence non-homothetic.
- *Marginal* propensity to consume is equal to one for all goods, which allows aggregation across households, hence we can talk about the representative household within each country. **Simple, but no effect of income distribution.**

Let us turn to some Ricardian models with non-homothetic preferences which do *not* allow such aggregation.

- Matsuyama (2000)
- Flam-Helpman (1987)
- Stokey (1991)

Matsuyama's (2000) Model of North-South Trade

Key Features:

- Goods are indexed according to priority. As their incomes go up, the households go down on their shopping list. The rich consume more variety of goods than the poor.
- *Asymmetric demand complementarities* across goods. Lower prices of high-priority (lower-indexed) goods increases demand for low-priority (high-indexed) goods, while lower prices of low-priority goods would not increase demand for high-priority goods.
- South (North) has comparative advantage in high-priority (low-priority) goods, hence specializing in goods with lower (higher) income elasticities of demand.

Main Results:

- The ToT move against South and ***product cycles*** occur due to a faster population growth in South, a uniform productivity gains in South, and a global productivity gains.
- The welfare gains of productivity growth are unevenly distributed. North can capture all the benefits of its own uniform productivity growth, while South may lose from its own uniform productivity growth. ***(Immiserizing Growth)***
- South's *domestic* policy, which distributes income from the rich to the poor shifts the demand composition towards its own goods and hence, improve its terms of trade. This effect could be so large that *every* household in South may be better off at the cost of North. ***(Transfer Paradox)***

Basic Model

Two Countries: Home (South) and Foreign (North)*. Foreign Labor as the *numeraire.*

Technology: A continuum of goods, $z \in [0,\infty)$.

(A1) $A(z) \equiv a^*(z)/a(z)$ is continuous and strictly decreasing in $z \in [0,\infty)$.

$p(z) = wa(z), z \in [0,m]$; $p(z) = a^*(z), z \in [m,\infty)$, where

(PT) $w = A(m)$.

As in Dornbusch-Fischer-Samuelson (DFS), except that *the goods space is open-ended.*

Households: N household at Home; N^* households at Foreign

- A Home household with h units of labor earns wh; h is distributed as $F(h)$.
- A Foreign household with h^* units of labor earns h^*; h^* is distributed as $F^*(h^*)$.

Preferences: A Household with income I maximizes

$$V = \int_0^\infty b(z)x(z)dz \text{, subject to } \int_0^\infty p(z)x(z)dz \le I \text{,}$$

$b(z)$: the utility weight attached to good z
$x(z)$: an indicator function, equal to 1 if good z is consumed and zero otherwise.

Note: Goods come in discrete units and each household's desire of a particular good satiates after one unit.

(A2) $b(z)/a(z)$ and $b(z)/a^*(z)$ are both strictly decreasing in z,

This ensures that $b(z)/p(z)$ is strictly decreasing so that **every household consumes goods in the same order given by (A1).**

- South has comparative advantage in high priority goods, which even the poor consume.
- North has comparative advantage in low priority goods, consumed only by the rich.

Consumption and Utility Measures:

Let $E(z) \equiv \int_0^z p(s)ds = \int_0^z \min\{wa(s), a^*(s)\}ds$ and $B(z) \equiv \int_0^z b(s)ds$.

A Home household with income, wh, consumes all the goods in $[0, u(h)]$ and enjoys the utility $V(h) = B(u(h))$, where $u(h)$ is given by

Home Household's Budget Constraint: $E(u(h)) = wh$.

A Foreign household with income, h^*, consumes all the goods in $[0, u^*(h^*)]$ and enjoys the utility $V^*(h^*) = B(u^*(h^*))$, where $u^*(h^*)$ is given by

Foreign Household's Budget Constraint: $E(u^*(h^*)) = h^*$.

Note: $B(z)$ is a one-to-one mapping, so $u(h)$ & $u^*(h^*)$ may be viewed as utility measures.

Because each household whose income satisfies $I \geq E(z)$ consumes one unit of good z,

Aggregate Demand for good z: $Q(z) = N[1- F(E(\text{z})/w)] + N^*[1- F^*(E(\text{z}))]$.

***Labor Market Equilibriums and the Balanced Trade*:**

Home Labor Market: $$L = N\int_0^\infty h dF(h) = \int_0^m a(z)Q(z)dz$$

$$\rightarrow \quad wL = wN\int_0^\infty h dF(h) = N\int_0^\infty \min\{wh, E(m)\}dF(h) + N^*\int_0^\infty \min\{h^*, E(m)\}dF^*(h^*).$$

Note: a Home household with h spends min$\{wh, E(m)\}$ and a Foreign household with h^* spends min$\{h^*, E(m)\}$ on the Home goods.

Foreign Labor Market: $$L^* = N^*\int_0^\infty h^* dF^*(h^*) = \int_m^\infty a^*(z)Q(z)dz$$

$$= N\int_0^\infty \max\{wh - E(m), 0\}dF(h) + N^*\int_0^\infty \max\{h^* - E(m), 0\}dF^*(h^*)$$

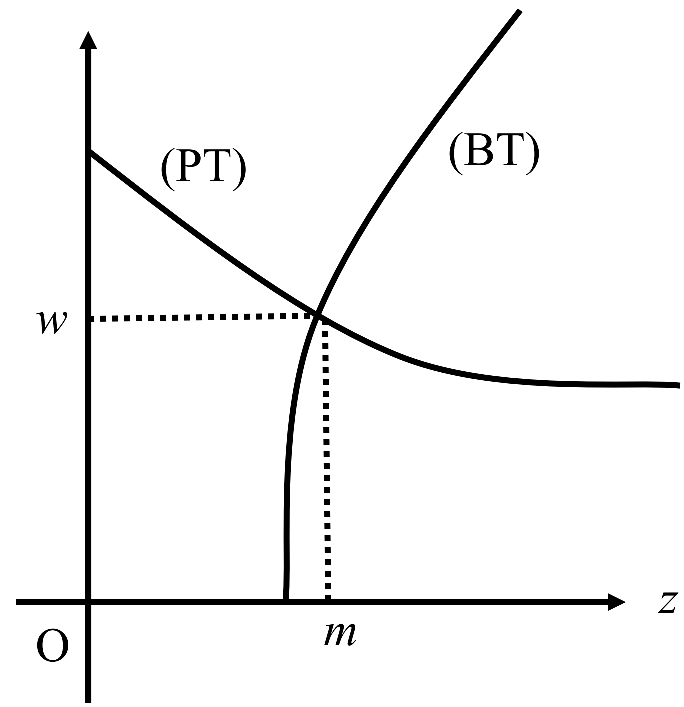
Note: Due to Walras's Law, two Labor Market Equilibriums are equivalent, which can be further rewritten as the Balanced Trade Condition (BT).

(BT) $$N\int_0^\infty \max\{h - \int_0^m a(s)ds, 0\}dF(h) = N^*\int_0^\infty \min\{\frac{h^*}{w}, \int_0^m a(s)ds\}dF^*(h^*).$$

(PT) and (BT) jointly determine m and w.

(BT) is upward-sloping, as long as some Foreign households are poor enough to consume only the Home goods.

If w is sufficiently small that all the Foreign households are rich enough to consume some Foreign goods, a small change in w does not affect the demand for Home labor. Hence, (BT) is vertical at m, satisfying $(N+N^*)\int_0^m a(s)ds = L$.



***A Comparison with the Dornbusch-Fischer-Samuelson (DFS) model*:**

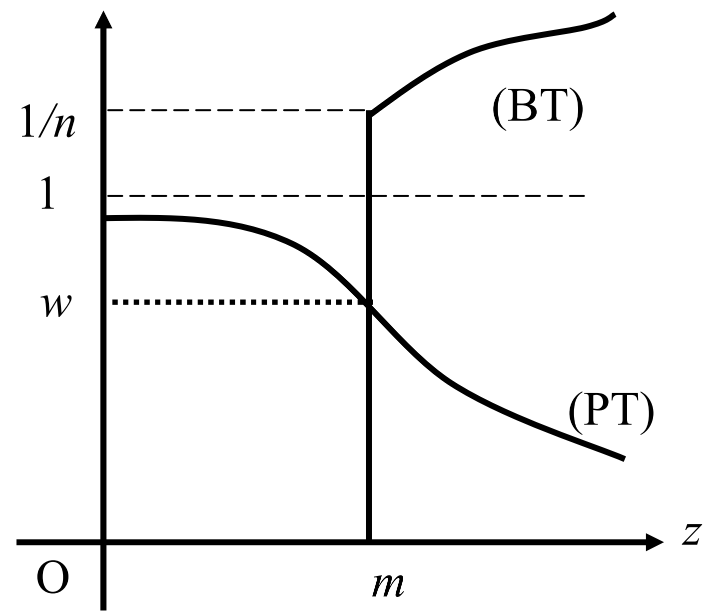
• (BT) depends on $F(h)$ and $F(h^*)$ here, but not in DFS.
• Here, the effects of L or L^* depend on whether they come from changing N or N^*, holding $F(h)$ and $F(h^*)$ constant, or changing $F(h)$ and $F(h^*)$, holding N or N^* constant.
• In DFS, (BT) goes to the origin. As the Home labor and Home goods become cheaper, demand for Home labor increases through *substitution effects*. To keep the Home labor market in equilibrium, Home's production shifts toward the bottom end of the goods spectrum. Here, as $w \to 0$ along (BT), m approaches a positive number, given by $(N+N^*)\int_0^m a(s)ds = L$. Demand for Home labor does not increase, when the Home goods prices go down. The total demand for each good is bounded by $N+N^*$. Home must continue producing a certain range of goods to keep all the Home labor employed.
• In DFS, $a(z)$ and $a^*(z)$ do not appear in (BT), due to Cobb-Douglas. Here, they appear asymmetrically. Reducing $a(z)$ and hence the Home goods prices shifts the spending *away from* Home goods toward Foreign goods, leading to higher relative demand for Foreign labor. To restore the balance, Home must expand its range of production. On the other hand, $a^*(z)$ does not appear in (BT), because a reduction in $a^*(z)$, and Foreign goods prices only induce the household to buy other Foreign goods with higher indices, and hence does not cause a spending shift between Home and Foreign goods.

North-South Trade: Homogeneous Populations

- $h = h^* = 1$ for all households and hence $N = L$ and $N^* = L^*$. Denote $n \equiv N/(N+N^*)$.
- $A(z) < 1$ for all z, so that Foreign (North) is richer than Home (South), $w < 1$.

(BT) $\int_0^m a(s)ds = n$ (for $w < 1/n$).

(PT) $w = A(m)$.



Utilities (and the ranges of goods consumed), u and u^*, satisfy $m < u < u^*$ and are given by

$$E(u) = w\int_0^m a(s)ds + \int_m^u a^*(s)ds = w;$$

$$E^*(u^*) = w\int_0^m a(s)ds + \int_m^{u^*} a^*(s)ds = 1,$$

- $u < u^*$ because North is richer than South.
- $m < u$ because North imports $z \in [0,m]$ from South; hence, South must also imports something from North to keep the trade balanced.

Some Comparative Statics

Relative Population Sizes:

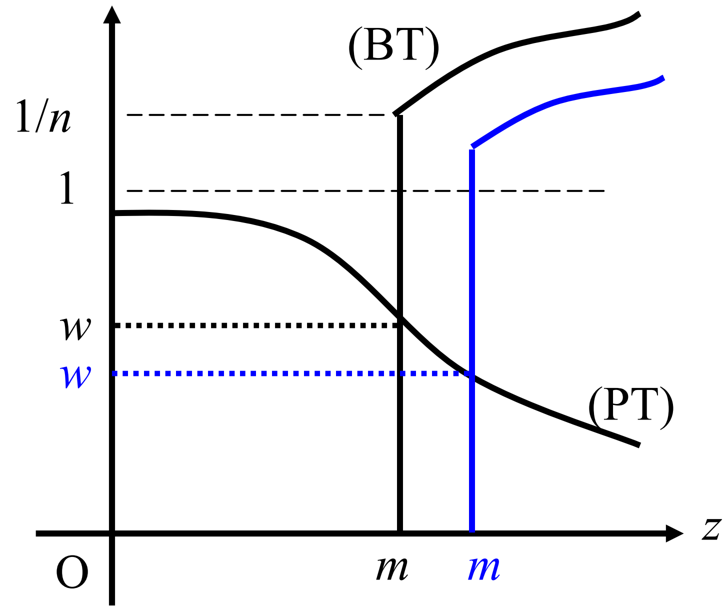
$n \uparrow \rightarrow m \uparrow$ and $w \downarrow$

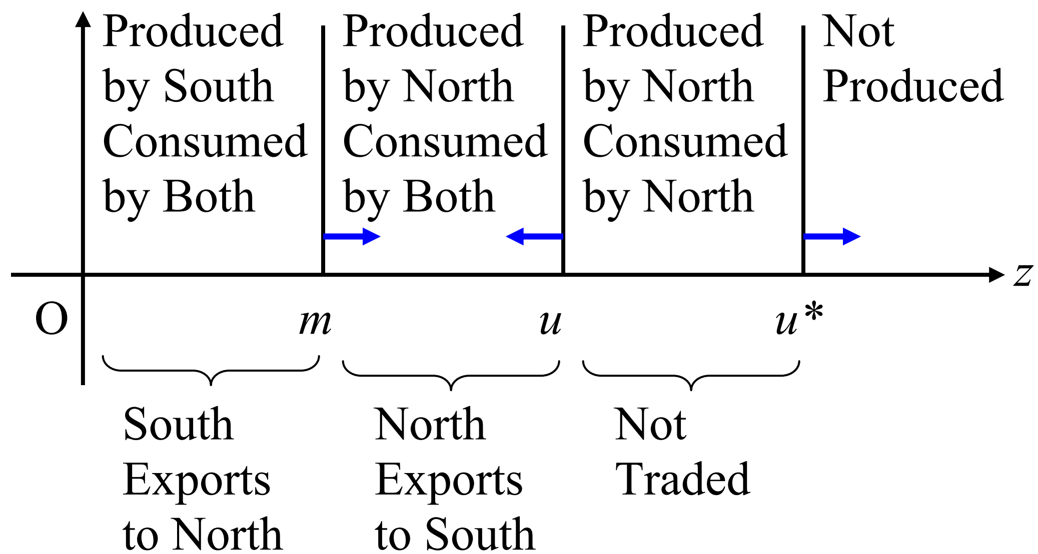
$$a(m)dm = dn > 0$$

$$dw = A'(m)dm < 0,$$

$$a^*(u)du = (1-n)dw < 0,$$

$$a^*(u^*)du^* = -ndw > 0.$$





Notes:

- The welfare effect is purely distributional, as $a^*(u)Ndu + a^*(u^*)N^*du^* = 0$.

- In DFS, the *vertical* distance of the (BT) curve depends on the relative country size. Hence, a higher n shifts (BT) *down*, causing a less-then-proportional decline in the Home relative wage. The Home share in the world income thus goes up. Here, the *horizontal* distance of the (BT) curve depends on the relative size. Hence, a higher n shifts (BT) *to the right*, which could cause a big ToT deterioration. Hence, the Home share in the world income may go down.

- If the population continues to grow faster in South than in North, South experiences a secular decline in its terms of trade, similar to Prebisch and Singer. The lower end of industries in North continuously migrate to South, and new industries are born continuously in the North, generating *Product Cycle* phenomena, similar to those discussed by Linder (1961) and Vernon (1966).

(Infinitesimal) Productivity Changes: $g(z) \equiv -\frac{\partial a(z)}{a(z)}$, $g^*(z) \equiv -\frac{\partial a^*(z)}{a^*(z)}$

$$a(m)dm = \int_0^m g(s)a(s)ds,$$

$$dw = A'(m)dm + w\{g(m) - g^*(m)\}$$

Welfare Implications:

$$a^*(u)du = w\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds + (1-n)dw$$

$$a^*(u^*)du^* = w\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds - ndw$$

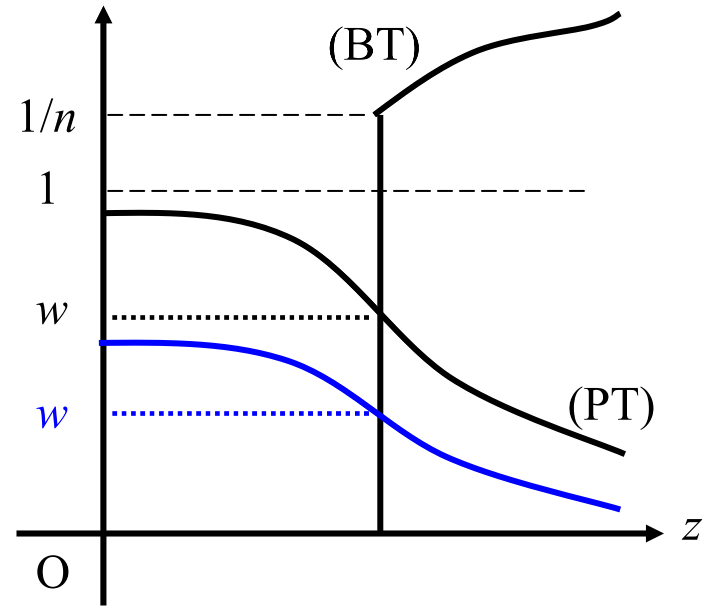
The last terms represent the terms of trade effect, which determines how the overall welfare gains, $a^*(u)ndu + a^*(u^*)(1-n)du^* > 0$, are distributed between North and South.

Northern productivity growth: $g(z) = 0$; $g^*(z) > 0$.

$dm = 0$; $\qquad dw/w = -g^*(m) < 0$;

$$a^*(u)du = -(1-n)wg^*(m) + \int_m^u g^*(s)a^*(s)ds$$
$$= \int_m^u \left\{g^*(s) - g^*(m)\right\}a^*(s)ds;$$

$du^* > 0$;



Uniform Case: $g^*(z) = g^*$ for all $z \in [m, u]$, $du = 0$.

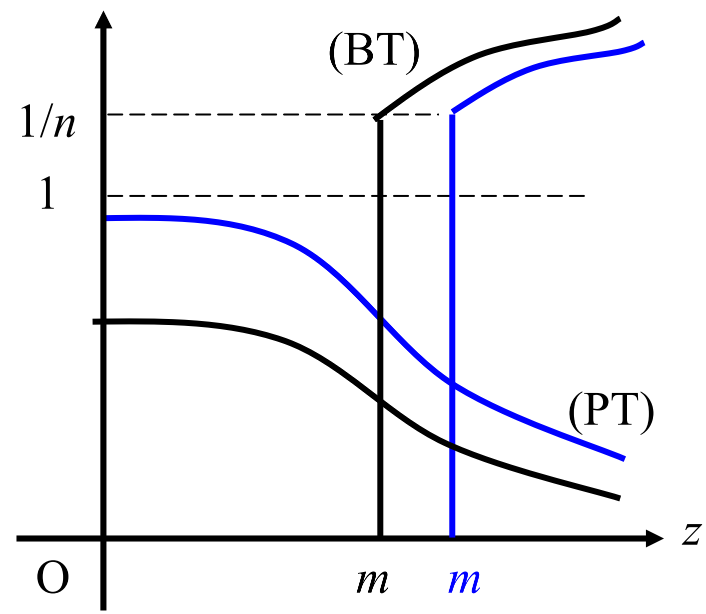
No spillover to South. A higher income of Northern households leads to more demand for the North goods. This is different from population growth in North, which leads to more demand for the South goods, hence benefits South.

Exercise: Examine the effects of increasing $h^* > 1$, while keeping $h = 1$. How is this different from the uniform productivity gains in North, discussed above?

Non-Uniform Case: du > (<) 0 if $g^*(z)$ is increasing (decreasing) over $[m, u]$.

South benefits when the change in North amplifies the existing patterns of comparative advantage, and loses otherwise.
Exercise: Discuss how this is different from Foreign non-uniform productivity gains in the DFS model.

Southern productivity growth: $g(z) > 0$; $g^*(z) = 0$.



$$a(m)dm = \int_0^m g(s)a(s)ds > 0$$

$$dw = A'(m)dm + wg(m)$$

$$a^*(u)du = (1-n)dw + w\int_0^m g(s)a(s)ds\ .$$

$$a^*(u^*)du^* = -\frac{nA'(m)}{a(m)}\int_0^m g(s)a(s)ds + w\int_0^m \{g(s)-g(m)\}a(s)ds$$

Uniform Case: g(z) = *g* for all $z \in [0,m]$,

$$a(m)dm = ng > 0; \qquad \frac{dw}{w} = \frac{A'(m)}{w}dm + g = \left[1 + \frac{nA'(m)}{wa(m)}\right]g < g$$

$$a^*(u)du = (1-n)dw + nwg = \left[w + \frac{n(1-n)A'(m)}{a(m)}\right]g; \qquad a^*(u^*)du^* = -\frac{n^2 A'(m)}{a(m)}g > 0\,.$$

- The terms of trade move in favor of North (since $dw/w < g$).
- The cheaper South goods allow the households in North to expand their consumption.
- ***Product Cycles*** emerge (the birth of new industries in North, $du^* > 0$, and the migration of some industries from North to South, $dm > 0$)
- The effects on w and u are ambiguous.

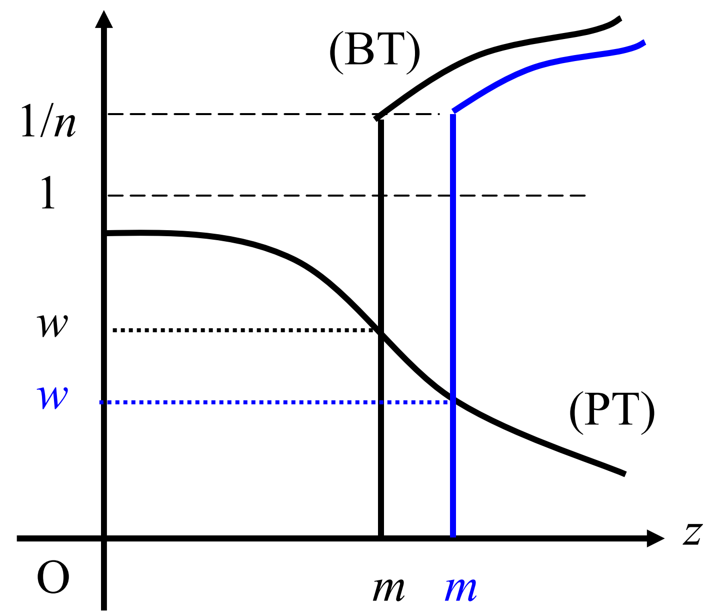
If $-A'(m) > a(m)w/n = a^*(m)/n$, the South's factor terms of trade deteriorates.
If $-A'(m) > a^*(m)/n(1-n)$, the deterioration is so large that $du < 0$; ***Immiserizing Growth.***

Exercise: Examine the effects of increasing $h > 1$, while keeping $h^* = 1$, which is small enough that $wh < 1$ continues to hold. How is this different from the uniform productivity gains in South discussed above?

Non-Uniform Case: $du^* < 0$, if $g(z)$ is sufficiently small over $[0,m]$, relative to $g(m)$.

- South captures more than 100% of all the world's productivity gains.
- North loses its industries at both ends of its spectrum.
- This situation may arise from Technology Transfers, as South has more to learn from North for higher-indexed goods.

Global productivity improvement: $g(z) = g^*(z) > 0$.



$$a(m)dm = \int_0^m g(s)a(s)ds > 0$$

$$dw = A'(m)dm < 0$$

$a^*(u)du$

$$= \left[w + (1-n)\frac{A'(m)}{a(m)} \right] \int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds$$

$$a^*(u^*)du^* = \left[w - n\frac{A'(m)}{a(m)} \right] \int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds > 0.$$

The effect on u is ambiguous, while $du^* > 0$ unambiguously.

In spite of the world-wide productivity gains, the asymmetry of demand response causes ToT to move against South and leads to Product Cycles (dm, $du^* > 0$).

Exercise: Examine the effects of ***Transfer Payments*** made from North to South, financed by lump-sum taxes in North, and distributed by lump-sum transfers in South.

Exericse:

In the above model, keep the first assumption:

- $h = h^* = 1$ for all households and hence $N = L$ and $N^* = L^*$.

but, change the second assumption to

- $A(z) > 1$ for all z, so that Foreign is poorer than Home to ensure $w > 1$.

Redo all the exercises discussed above under this alternative assumption.

Note: This may capture the situation where Home is the *Rich North*, which has comparative advantage in industrial goods consumed by the mass, while Foreign is the *Poor South*, which has comparative advantage in offering exotic Holiday Resorts, which only the rich people can afford.

North-South Trade: The Case of Heterogeneous Populations

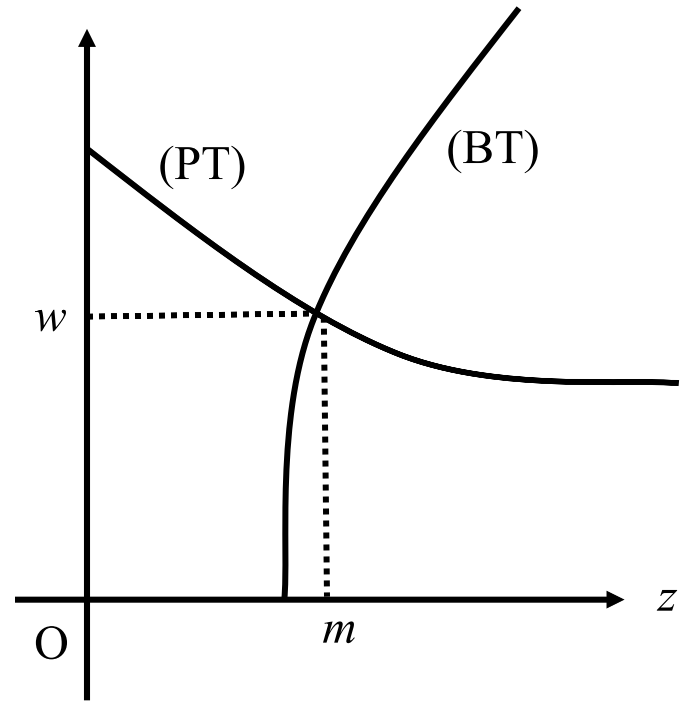
Let us assume:

- $F(h)$ and $F(h^*)$ are nondegenerate.

- Their supports include small h or h^*, such that, in equilibrium, $wh < E(m)$ or $h^* < E(m)$.

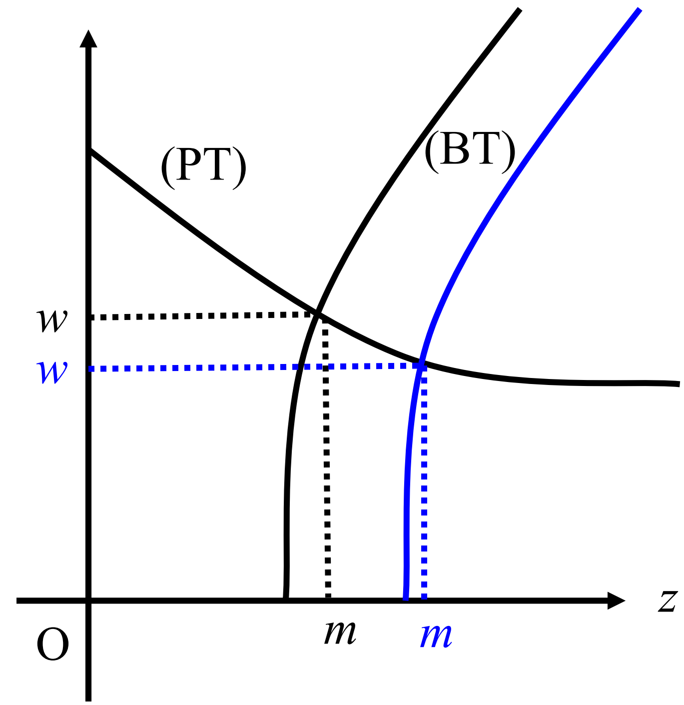
✓Some poor households do not consume goods produced in North: $u(h) < m$ or $u^*(h^*) < m$.
✓(PT) intersects at the upward-sloping part of (BT).

- The world's richest household, which determines the upper end of the North goods, resides in North.



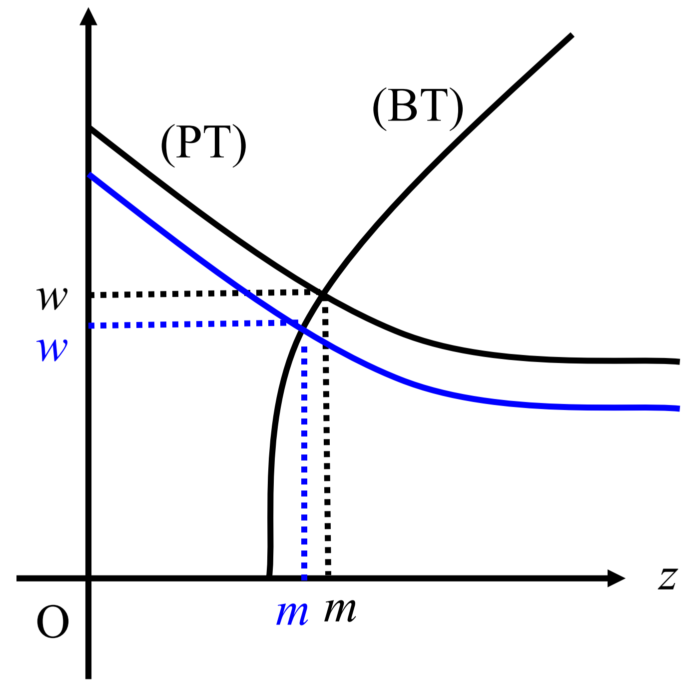
Relative Population Size: A faster population growth in South ($dn > 0$) shifts the (BT) curve to the right, hence $dm > 0$ and $dw < 0$.

- All households in North, the rich and the poor, are better off, as the ToT improves.
- New industries are born, as $dw < 0$ implies that the world's richest becomes richer, and these industries are in North
- Some old industries migrate from North to South ($dm > 0$): ***Product Cycles.***
- The rich in South, those with $u(h) > m$, are worse off, as the ToT moves against them.
- The poor in South, those with $u(h) < m$, are unaffected, as they essentially live in autarky, and hence are insulated from the ToT change.



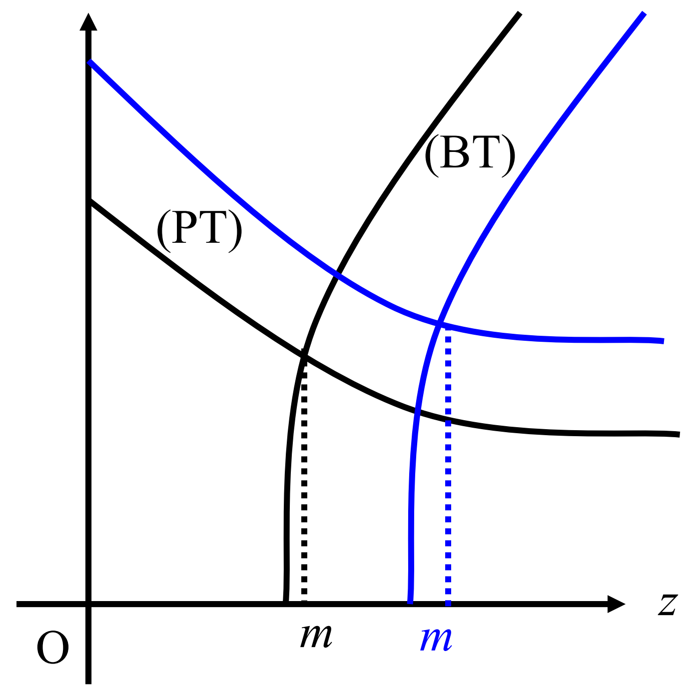
Productivity growth in North ($g(z) = 0$, $g^*(z) > 0$) shifts (PT) down, hence $dw < 0$.

- All households in North are better off.
- With (BT) upward-sloping, $dm < 0$, and $-g^*(m) < dw/w < 0$. As the ToT improves, the poor in North, those with $u^*(h^*) < m$, consume more South goods, increasing demand for South's labor. To keep its labor market in balance, South specializes in a narrower set of goods, abandoning the upper end of industries, which move to North.
- With $-g^*(m) < dw/w$, the rich in South, those with $u(h) > m$, are better off if $g^*(z)$ is constant over $(m, u(h)]$. If not, they can be worse off.
- The poor in South, those with $u(h) < m$, are unaffected.



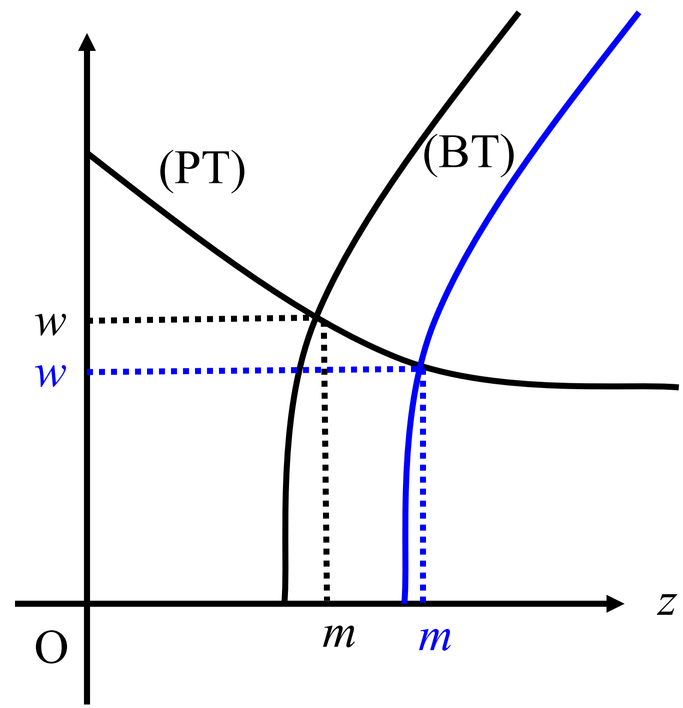
Productivity growth in South ($g(z) > 0$, $g^*(z) = 0$) shifts (PT) up and (BT) to the right, hence $dm > 0$, and $-\infty < dw/w < g(m)$.

- If $g(z)$ is constant over $[0,m]$, all households in North are better off; New industries are born in North. ***Product Cycles, again***.
- If $g(z)$ is faster at m than $[0,m)$, North can be worse off.
- The effect on the rich in South, those with $u(h) > m$, is ambiguous even with the uniform change. They can be worse off if the ToT moves against them.
- The poor in South, those with $u(h) < m$, insulated from the ToT change, are better off.



Global productivity growth ($g(z) = g^*(z) > 0$) shifts (BT) to the right, while (PT) unchanged, hence $dm > 0$ and $dw < 0$.

- All households in North are better off, and new industries are born. With $dm > 0$, ***Product Cycles*** again.
- The poor in South, those with $u(h) < m$, are better off.
- The effect on the rich in South, those with $u(h) > m$, is ambiguous.

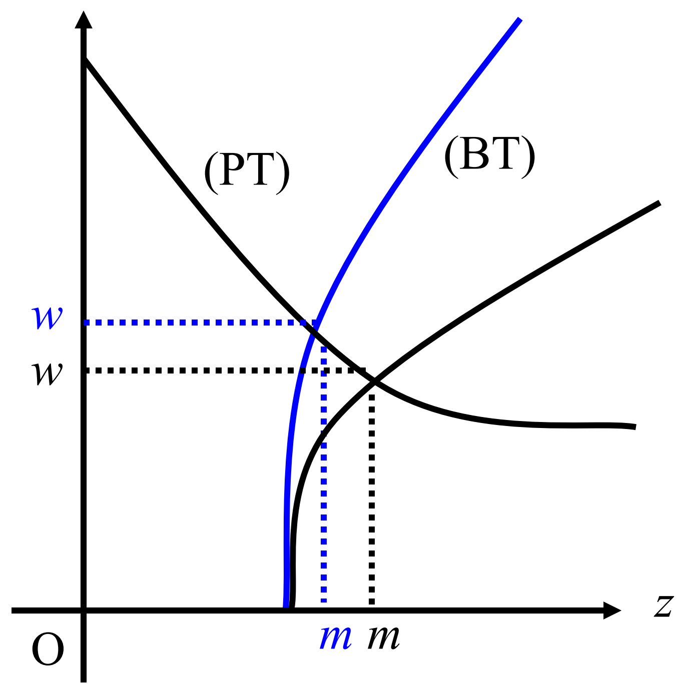


Income Transfers:

Consider South's *domestic* transfer policy, which redistributes income from the Rich, those with $u(h) > m$, who spend their additional income on imports from North, to the Poor, those with $u(h) < m$, who spend their additional income on the South goods.

(BT) shifts up, hence $dm < 0$ and $dw > 0$.

- All households in North are worse off, as the ToT moves against North.
- The poor in South are better off, due to the transfer; no effect from the ToT change.
- The rich in South: their income is taken away, but the ToT moves in favor. Perhaps, paradoxically, they may be better off.



Example:

- Homogenous households in North: $h^* = 1$.
- Two-types of households in South: 50% with h_L and 50% with h_H, where $h_L < h_H$.

With a transfer per household, T, measured in Home labor,

South's Labor Market: $w(h_L+h_H)N/2 = \{w(h_L+T)+ E(m)\}N/2 + N^*E(m)$,

→ $\int_0^m a(s)ds = (h_H - T)(2-n)/n$

→ $dw = A'(m)dm = -\dfrac{A'(m)}{a(m)}\left(\dfrac{2-n}{n}\right)dT > 0$, evaluated at $T = 0$,

Rich South Household's Budget Constraint: $w\int_0^m a(s)ds + \int_m^{u_H} a^*(s)ds = w(h_H - T)$.

→ $a^*(u_H)du_H = -wdT + \left[\int_m^{u_H} a^*(s)ds\right]dw.$

$$= \left\{-\frac{A'(m)}{a(m)}\left(\frac{2-n}{n}\right)\left[\int_m^{u_H} a^*(s)ds\right] - w\right\}dT$$

With a sufficiently large $|A'(m)|$, the positive ToT effect offsets more than the primary effect of transfer.

All the households in South may be better off by adopting a "domestic" policy of redistributing from the rich to the poor (at the expense of North).

Likewise,

- If the rich in South steal income from the poor in South, all the southern households can be made worse off, including the rich who exploit the poor. (North benefits)

- If North adopts a domestic policy of redistributing income from the rich to the poor, the resulting ToT deterioration can make all the households in North can be worse off, including the poor, who receives the transfer. (South benefits.)

Notes:

- This may be viewed as an example of 3-Agent Transfer Paradox: see Bhagwati-Brecher-Hatta (1983). Indeed, one may also interpret this example as a 3-country model with High-income North, Middle-income South, and Low-income South, where the population is homogeneous within each country, and Middle-income South and Low-income South differ only in their labor endowments.

- A close connection between this result and the earlier result of Immiserizing Growth, which states,

✓The poor South, who nevertheless is rich enough to buy goods from North, may lose from its own productivity growth, as this could cause a large ToT deterioration.
✓The flip side of Immiserizing Growth is that they could gain from throwing away some of their income.
✓Here, instead of throwing away, they give it to the poorest who do not affect the ToT.

- An extension of this model to a multi-country setting is just a short-step from the above example: see Matsuyama (2000, Section V).

- Stibora-de Vaal (2007) studied the effects of trade policies in this model.

Flam-Helpman (1987): Vertical Differentiation & North-South Trade

Like my (2000) model,

- A continuum of goods; the set of goods produced is endogenous.
- Only the rich demand for higher-indexed goods. As the households become richer, new goods are introduced at the upper end.
- North (South) has comparative advantage in higher (lower)-indexed goods.

Unlike my (2000) model,

- Goods are indexed according to product quality, and high-quality and low-quality goods are gross substitutes.
- A reduction in the prices of a low quality good induces the households to *switch* from high quality to low quality good.
- Some goods at the bottom end are not produced.

Interpretation: The goods are *vertically differentiated* products within an industry, and the model is used to address the issues of *intra-industry* trade.

Note: Nondegenerate income distributions are essential to generate intra-industry trade, as we need some poor households in North, who buys low-quality southern goods, and some rich households in South, who buy high-quality northern goods.

Some Main Results:

- Technical progress and population growth brings the introduction of high quality goods and the disappearances of low quality goods.
- Goods in the middle are not produced.
- A shift that causes a continuing deterioration of South's terms of trade, which makes South goods cheaper, causes some goods to disappear from North and reemerge in South, but only with some delay.
- A deterioration of South's terms of trade also discourages North from producing the upper end of the spectrum.

The Model

Two Countries: Home (North) and Foreign (South)*. Foreign Labor as the *numeraire.*

Two Types of Goods:

- *Outside Good*, y, which may be produced and consumed in any quantity
- *Vertically Differentiated Products*, $z \in [0,\infty)$, which comes in discrete units

Technologies:

For the outside good, $a = a^* = 1$. For the vertically differentiated products,

(A1) $A(z) \equiv a^*(z)/a(z) > 1$ is continuous and strictly increasing in $z \in [0,\infty)$.

$p(z) = a^*(z)$, $z \in [0,m]$; $p(z) = wa(z)$, $z \in [m,\infty)$, where

(PT) $w = A(m) > 1$,

ensuring that South produces the outside good, whose price is equal to one.

Households: N household at Home; N^* households at Foreign

- A Home household with h units of labor earns wh; h is distributed as $F(h)$.
- A Foreign household with h^* units of labor earns h^*; h^* is distributed as $F^*(h^*)$.

Preferences: A Household with income I chooses y, the *quantity* of the outside good, and z, the *quality* of the vertically differentiated product, to

$$\text{Max } u(y, z) \qquad \text{subject to } y + p(z) \leq I.$$

Note: The desired quantity of the vertically differentiated product is assumed to be one.

We want to ensure that high income households choose a higher z, which can be interpreted as high-quality, so that, when, combined with (A1),
- South has comparative advantage in low-quality products, chosen by the poor.
- North has comparative advantage in high-quality products, chosen by the rich.

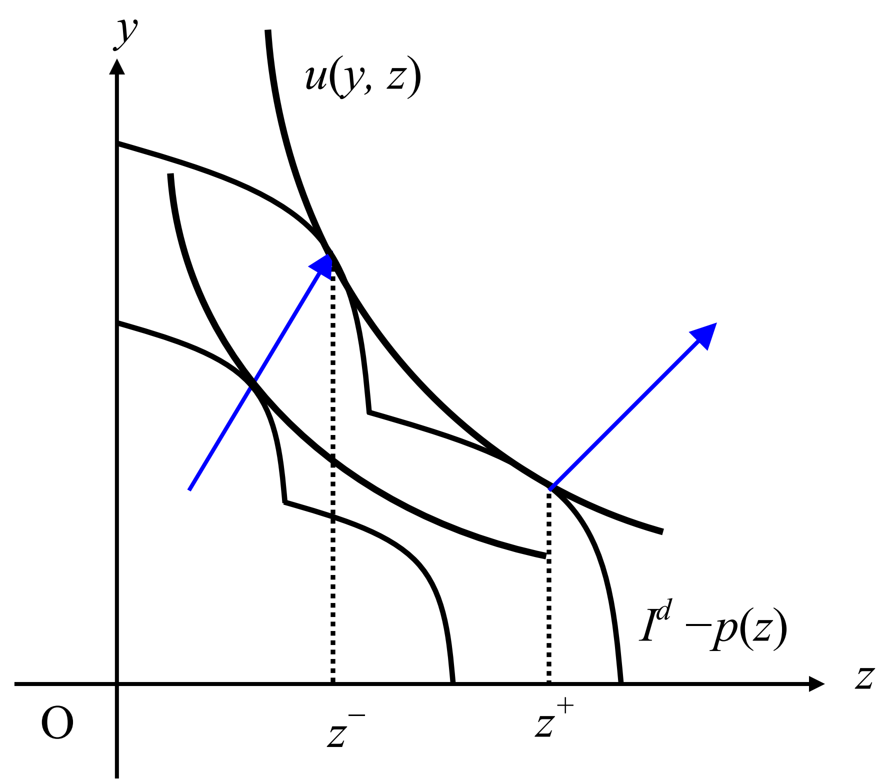
Flam-Helpman work with the specific functional forms:

$$u(y, z) = ye^{\alpha z}; \qquad a(z) = e^{\gamma z} / A; \; a^*(z) = e^{\gamma^* z} / A^*, \quad \text{with} \qquad \alpha > 0; 0 < \gamma < \gamma^*.$$

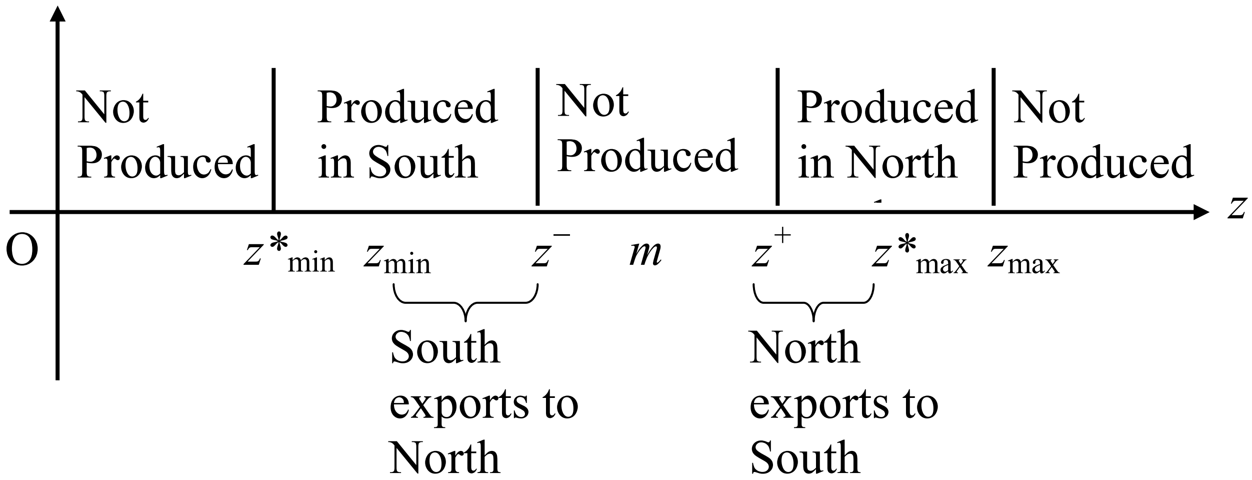
The solution to the maximization yields an income level, I^d, such that

- Households with $I = I^d$ are indifferent between $z^- < m$, and $z^+ > m$.
- Those with $I < I^d$ buy low-quality South goods $z < z^-$.
- Those with $I > I^d$ buy high-quality North goods, $z > z^+$.

Figure 1 of Flam-Helpman



Equilibrium Patterns of Production and Trade:



$z_{\min}$ ($z^*_{\min}$): chosen by the poorest households in North (South)
$z_{\max}$ ($z^*_{\max}$): chosen by the richest households in North (South).

Intra-industry trade takes place if there are some households in North with $I < I^d$ (hence, $z_{\min} < z^-$) and some households in South with $I > I^d$ (hence, $z^*_{\max} > z^+$).

Flam-Helpman (1987) conducted comparative statics on income distributions, relative population sizes, productivity growth, etc.

Stokey's (1991) Model of Vertical Differentiation and North-South Trade

Flam-Helpman has the properties that

- Each household must choose only one from a continuum of vertically differentiated goods. The rich who wear expensive evening gowns will not wear T-Shirts.
- Unless the supports of income distributions overlap, no intra-industry trade between North and South.

Stokey (1991) applied her (1988) model of vertical differentiation to North-South trade.

- Higher-quality goods offer more desirable features than lower-quality goods. (Cheap clothes only help you stay warm. Expensive cloths not only help you stay warm but also help you look good.) More specifically,

A continuum of features, $\xi \in [0,\infty)$ over which preferences are defined.
A continuum of goods z $\in [0,\infty)$; Good z offers one unit of all the features, $\xi \in [0,z]$.
A household with income I maximizes, given the prices of good z, $p(z)$,

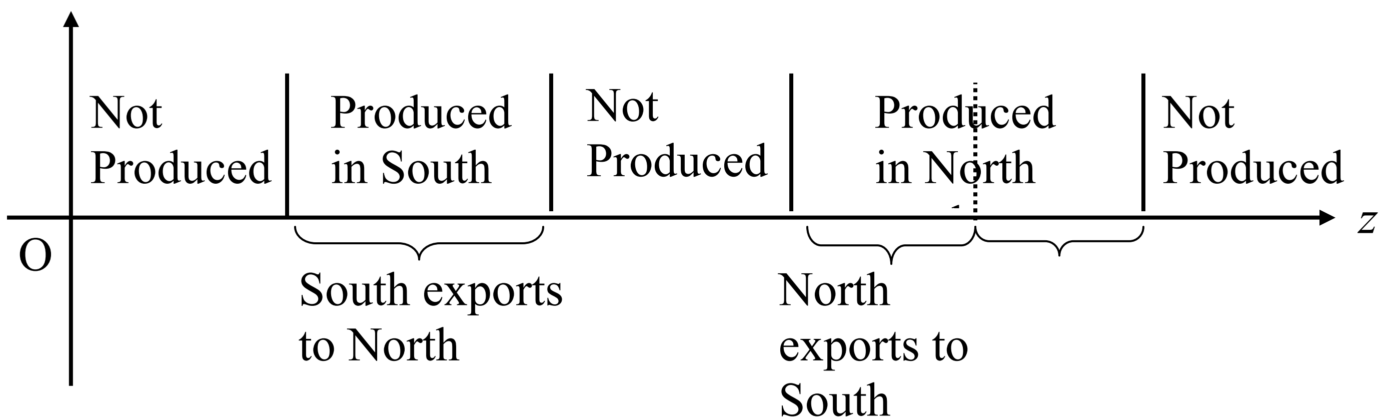
$$V = \int_0^\infty u(q(\xi))d\xi, \text{ s.t. } q(\xi) = \int_\xi^\infty c(z)dz \text{ and } \int_0^\infty p(z)c(z)dz \le I,$$

where $q(\xi)$ is the units of feature ξ consumed and $c(z)$ is the units of good z purchased.

- The rich may want to buy a range of vertically differentiated goods, both high and low quality, while the poor may be able to afford only low-quality goods (unlike Flam-Helpman, more similar to my 2000 model).

- Even if the population is homogeneous within each country (so every household in North is strictly richer than every household in South), which she assumes, intra-industry trade may occur (unlike Flam-Helpman, more similar to my 2000 model).

Equilibrium Patterns of Production and Trade



In spite of these differences between Flam-Helpman and Stokey, the two models share many similar properties.

- Technical progress and population growth brings the introduction of high quality goods and the disappearances of low quality goods.
- Goods in the middle are not produced.
- A shift that causes a continuing deterioration of South's terms of trade, which makes South goods cheaper, causes some goods to disappear from North and reemerge in South, but only with some delay.
- A deterioration of South's terms of trade also discourages North from producing the upper end of the spectrum.

Personal Note: I do not know what to make of the feature of the Flam-Helpman-Stokey models that there is always a gap in the middle.

Some Research Ideas:
- What if there are many industries that are vertically differentiated as in the Flam-Helpman or Stokey models?
- Would it be feasible (and interesting) to consider a hybrid of models similar to Flam-Helpman-Stokey and one similar to my 2000 model?

Multi-Country Extensions and Bilateral Trade

It is relatively straightforward to extend a two-sector Ricardian model to a multiple country setting: see Becker (1950) and Yanagawa (1996).

Two-Sector, Ricardian Model with a *Continuum* of Countries, $c \in [0,1]$.

Country $c \in [0,1]$ is characterized by

Labor Endowment: L^c
Unit Labor requirements: (a_1^c, a_2^c)
Expenditure Function (identical across countries): $E^c(p,u) = e(p)u$

With little loss of generality, assume $A^c \equiv a_1^c / a_2^c$ is strictly decreasing in c.

Exercise: Characterize the world economy equilibrium. How would some exogenous changes taking place in a country in the middle affect countries at the higher and lower ends?

How can develop a tractable multi-country, multi-sector Ricardian model?

Eaton and Kortum (2002): Technology, Geography, and Bilateral Trade

EK proposed a probabilistic formulation of technological heterogeneity that allows for
- a continuum of tradeable goods
- a finite number of countries
- trade costs that vary across country pairs but not across goods (so that they should be interpreted as "distances" or "geographical barriers")

Their model generates a very parsimonious relationship between the bilateral flows and three parameters representing
- *Absolute Advantage*, which determines the factor price differences
- *Comparative Advantage*, which promote trade
- *Geographical Barriers*, which restrict trade

They use this relation to estimate these parameters and quantify a variety of counterfactual experiments, such as
- the gains from trade
- study how technology and geography determine specialization
- the role of trade in spreading the benefit technology
- the consequences of tariff reductions

Probabilistic Approach to Technological Heterogeneity
(Here, I *try* to use the notations similar to the previous models to keep the continuity, rather than following Eaton-Kortum's).

A Finite Number of Countries, indexed by *c, d,* etc.

A Continuum of Tradeable Goods, indexed by $z \in [0,1]$.

Symmetric CES Preferences: $U \equiv \left\{ \int_0^1 [x(z)]^{1-1/\sigma} dz \right\}^{\sigma/(\sigma-1)}$

Iceberg Trade Costs: $\tau_{dc} > 1$.
When $\tau_{dc} > 1$ units of the good is shipped from the origin country, c, to the destination country, d, one unit of the good arrives. It is assumed that

- $\tau_{cc} = 1$ for each c
- $\tau_{dk}\,\tau_{kc} \geq \tau_{dc}$ (Triangle Inequality)

Note: Trade costs depend on *country-pairs*, but not on goods. Hence, they should be interpreted as *geographical barriers*, rather than the transportability of goods.

One Factor (Labor), whose price is denoted by w_c.

Production Technologies: $A_c(z)$ is *Labor Productivity* of Sector z in Country c.

Prices:

- Price of good z if shipped from c to d: $P_{dc}(z) = \dfrac{\tau_{dc} w_c}{A_c(z)}$
- Price of good z in country d: $P_d(z) = Min_c \{P_{dc}(z)\}$

Note: Triangular inequality means that the goods are never shipped indirectly.

Key Assumption**:** *Labor Productivity* as a random draw from a Frechet distribution,

$$\Pr[A_c(z) \le A] = F_c(A) = e^{-T_c A^{-\theta}}$$

- Distributions are independent across goods:
- With a large number (a continuum) of goods, $F_c(A)$ is also the fraction of the goods for which country's labor productivity is less than or equal to A.
- Distributions are also independent across countries.

- $T_c > 0$ governs the level of technology in country *c*. A higher T_c shifts the distribution in the sense of first-order stochastic dominance: *Absolute Advantage*.
- A higher θ means less variability: more incentive to trade with a lower θ, controlling the force of *Comparative Advantage.*

Note: Consider the two countries, Home and Foreign, whose labor productivities are randomly drawn as in Eaton-Kortum, with the parameters, T, T^*, and θ, so that the cdf's unit labor requirements are $F(a) = 1 - e^{-Ta^{\theta}}$ and $F^*(a^*) = 1 - e^{-T^*(a^*)^{\theta}}$, respectively. Then, $\Pr[a^*/a \geq \mathcal{A}] = z$ can be given by

$$\mathcal{A} = \mathcal{A}(z) \equiv \left(\frac{T}{T^*}\right)^{1/\theta} \left(\frac{1-z}{z}\right)^{1/\theta}.$$

This corresponds to $A(z)$ in the notation of DFS. A change in T or T^* causes uniform productivity gains. E-K specification has no room for biased productivity gains discussed earlier. (It has no room for non-homothetic preferences, either.)

Price Distributions:

- Price Distribution of good z if shipped from c to d:

$$G_{dc}(p) \equiv \Pr[P_{dc}(z) \le p] = \Pr\left[\frac{\tau_{dc}w_c}{p} \le A_c(z)\right] = 1 - F_c\left(\frac{\tau_{dc}w_c}{p}\right) = 1 - e^{-\left[T_c(\tau_{dc}w_c)^{-\theta}\right]p^{\theta}}$$

- Price Distribution of goods purchased in d:

$$G_d(p) \equiv 1 - \prod_{c=1}^{C}[1 - G_{dc}(p)] = 1 - e^{-\Phi_d p^{\theta}}, \text{ where } \Phi_d \equiv \sum_{c=1}^{C} T_c(\tau_{dc}w_c)^{-\theta}.$$

Notes:

- In the absence of trade costs (a zero-gravity world), $\tau_{dc} = 1$ for all c, the law of one price holds for each good, and hence Φ_d and $G_d(p)$ are independent of d.
- In autarky, $\tau_{dc} \to \infty$ for $c \neq d$, and hence $\Phi_d = T_d(w_d)^{-\theta}$.
- A remote country (d such that τ_{dc}'s are higher) has a lower Φ_d, and hence higher prices.
- Given the CES preferences, the exact price index, $e(p)$, in country d is given by

$$p_d \equiv \left\{\int_0^1 [p(z)]^{1-\sigma} dz\right\}^{1/(1-\sigma)} = \left\{\int_0^{\infty} p^{1-\sigma} dG_d(p)\right\}^{1/1-\sigma} = \gamma(\Phi_d)^{-1/\theta},$$

for $\sigma < 1 + \theta$, where γ, which depends on σ, is constant across countries.

Frechet is convenient because it is close under the min operation. Furthermore, it implies

- Probability that c is the lowest cost supplier of good z to d:

$$\pi_{dc} \equiv \int_0^\infty \prod_{s \neq c} [1 - G_{ds}(p)] dG_{dc}(p) = \frac{T_c(\tau_{dc} w_c)^{-\theta}}{\Phi_d} = T_c \left(\frac{\gamma \tau_{dc} w_c}{p_d} \right)^{-\theta},$$

which is also equal to the fraction of the goods that d purchases from c.

- Price Distribution of goods that d purchased from c is to $G_d(p) = 1 - e^{-\Phi_d p^\theta}$. It is independent of c. Thus, *d's* expenditure per good does not vary across countries. This means that country *c's* share in the country *d's* expenditure is also equal to the fraction of the goods that d purchases from c:

$$\frac{E_{dc}}{E_d} = \pi_{dc} = \frac{T_c(\tau_{dc} w_c)^{-\theta}}{\Phi_d} = T_c \left(\frac{\gamma \tau_{dc} w_c}{p_d} \right)^{-\theta}.$$

Intuition: A country with a lower T_c, or a higher w_c, or a higher τ_{dc}, sells a narrower range of goods to the destination, d. But, if you just look at the country's goods sold in *d,* they sell each good on the average by the same amount at the same price as any other countries. All variations across countries come at the extensive, not intensive, margin.

Real Wage (and GDP per capita): Setting d = c for the expression of π_{dc} yields

$$\frac{w_c}{p_c} = \frac{1}{\gamma}\left(\frac{T_c}{\pi_{cc}}\right)^{1/\theta} > \frac{1}{\gamma}(T_c)^{1/\theta} = \left(\frac{w_c}{p_c}\right)^{Autarky}, \quad \text{because } \pi_{cc} = 1 \text{ in autarky.}$$

Labor Market Equilibrium:

$$w_c L_c = \sum_{d=1}^{C} E_{dc} = \sum_{d=1}^{C} \pi_{dc} E_d = \sum_{d=1}^{C} \pi_{dc} w_d L_d = T_c \sum_{d=1}^{C} \frac{(\tau_{dc} w_c)^{-\theta}}{\sum_{s=1}^{C} T_s (\tau_{ds} w_s)^{-\theta}} w_d L_d \,,$$

which give the $C - 1$ independent conditions, which can be solved *numerically* for the equilibrium wage vector, determining the $C - 1$ relative wages.

- For the discussion about the uniqueness, see Alvarez and Lucas 2007).
- In the zero-gravity case ($\tau_{dc} = 1$ for every country pairs), we can solve this explicitly:

$$w_c = \kappa\left(\frac{T_c}{L_c}\right)^{1/(1+\theta)}; \qquad \frac{w_c}{p_c} = \frac{1}{\gamma} T_c^{\,1/(1+\theta)} \left[\sum_{m=1}^{C} \left[T_m \left(\frac{L_m}{L_c}\right)^{\theta}\right]^{1/(1+\theta)}\right]^{1/\theta},$$

which shows the properties similar to the effects of relative population size and uniform productivity changes discussed in the DFS.

Bilateral Trade Flows and Gravity:

$$\frac{E_{dc}}{E_c} = \frac{E_{dc}}{\sum_{m=1}^{C} E_{mc}} = \frac{(\tau_{dc}/p_d)^{-\theta} E_d}{\sum_{m=1}^{C} (\tau_{mc}/p_m)^{-\theta} E_m}.$$

With the budget constraint, $E_c = Y_c$,

$$E_{dc} = \frac{1}{\sum_{m=1}^{C} (\tau_{mc}/p_m)^{-\theta} Y_m} \frac{Y_c Y_d}{(\tau_{dc}/p_d)^{\theta}}.$$

The bilateral trade flows are proportional to the product of the two countries' GDPs (the masses) and inversely related to "the distance" to the power of θ.

- "The distance" here is defined by τ_{dc}/p_d, not by τ_{dc}. Its proximity to Germany, d, gives Czech, c, some advantages in exporting to Germany. However, many other countries are also close to Germany, whose effect is captured by p_d, which offset Czech's advantages.

- The distance has smaller effects with a smaller θ. With more technological variations across countries, there is better chance that exporters can penetrate into a remote market.

- "The distance" here affects only the trade costs, but not technological heterogeneity. If Italy and Spain (or Finland and Sweden) have similar technologies with each other, one should expect more trade between Italy/Spain and Finland/Sweden than the model predicts.

Note: What is shown here is a special case of E-K, who assumed

- The above Ricardian structure is applied only to the manufacturing sector (with α being the share of the manufacturing goods in the consumer expenditure).
- Producing each manufacturing good requires a Cobb-Douglas composite of labor and the composite of the manufacturing goods (with β being the share of the labor in the production of a manufacturing good).
- Here, we set $\alpha = \beta = 1$. Making β smaller magnifies the effects of trade costs.

Endogenous Technologies:

All the Ricardian models so far treat the technological heterogeneities as exogenous. Many new, and fascinating, issues arise if when we try to endogenize them. We will look at some examples in Part IV and in Part V.

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